Optimal Initial Public Offering design with aftermarket trading.*

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Abstract

This paper characterizes an optimal IPO in the presence of distinct adverse selection problems: one affecting the pre-market and the other affecting the aftermarket. We show that after market trading reduces informational rents but also generates some aftermarket rents when the shares are dispersed among heterogeneous investors. When informed clients can only gather informational rents, the optimal IPO resembles that of Benveniste and Spindt (1989) with the particularity that dispersion is optimal when aftermarket rents are low enough. But when these clients can also extract aftermarket rents, it is optimal to ration them for some values of the parameters and dispersion can be used to compensate clients with high appraisals for the stock.

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1 Introduction

The considerable amount of literature surrounding initial public offerings (IPOs) shows that the allocation and the underpricing of shares are real conundrums. Benveniste and Spindt (1989), a cornerstone in the literature, points at the presence of informational asymmetry between institutional investors and the underwriter. It explains that shares must be underpriced to induce informed investors to reveal their true estimation of the stock and the allocation must favour investors reporting high appraisals to minimize the necessary underpricing. These results are not always supported empirically. If one considers in particular the allocation of shares, the empirical literature has not even reached a consensus. According to Cornelli and Goldreich (2001) most of the bulk goes to institutional investors who participate regularly and to those who provide more information. Jenkinson and Jones (2004) argues instead that it goes to investors perceived to be long-term holders. One reason for the divergence in empirical results is the fact that the value of the shares partly depends on how shares are priced, how they are allocated, and what investors do with these shares. Thus, when designing an IPO the underwriter is, to some extent, determining the value of what he is selling. Benveniste and Spindt (1989) implicitly assumes that all investors are long-term holders and the value of the stock reflects their overall appraisals. Doing so it fails to address the simultaneity problem that is specific to IPO design. In this paper we attend to this issue as we characterize the offer prices and allocations of shares that maximize the IPO proceeds considering an element neglected so far: the aftermarket activity related to the newly issued shares.

The setting considered follows from Benveniste and Spindt (1989) in many ways. An underwriter is endowed with a stock of newly issued shares. His objective is to maximize the proceeds of the IPO. He faces both, informed and uninformed investors. The expected value of the stock reflects the informed investors’ overall interest. While Benveniste and Spindt (1989) allows for some residual uncertainty, it plays no role in their analysis as investors are long-term holders. Instead, we consider that investors may flip their share in the aftermarket. While some do so for liquidity reasons, others have access to some information and are able to trade strategically. Evidence that part of the aftermarket trades are based on private information can be found in Krigman et al. (1999) and more recently in Boehmer et al. (2006). We model the aftermarket as a competitive dealership market as in Glosten and Milgrom (1985). For tractability we consider that the probability of getting information in the aftermarket is the same for all (previously) informed investors and it is the same for all (previously) uninformed investors. However we consider that either of those two groups has an informational advantage. Thus, when the shares are dispersed (meaning that both informed and uninformed investors are served) those who are more likely to learn additional information extract rents in the aftermarket.
Due to the IPO’s inherent features, aftermarket trading triggers some costs and benefits. The dispersion of shares among heterogeneous investors generates aftermarket rents. The investors who can extract those rents submit more competitive bids. However, due to the uniform pricing rule the offer price is at most equal to the lowest winning bid and cannot be set so as to seize these rents. They end up forming part of the money left on the table. However, aftermarket trading also benefits the underwriter as it can facilitate truthful information revelation from investors during the IPO. Although the value of the stock ultimately reflects the informed investors’ overall interest, it often takes time for all information to be absorbed and correctly interpreted. In that light, we consider that when the market first opens it is the information incorporated in the offer-price that is used by the dealers when setting their bid prices. In other words, dealers infer the revealed information and not necessarily the investors’ true private information. Therefore, misreporting high interest triggers a negative externality for investors as it leads to a lower bid price. Thus, the more likely an investor is to flip the share in the aftermarket the more reluctant he is to downgrade his interest for the stock.

Our contributions in terms of the optimal IPO are the following. The money left on the table consists of informational rents as well as aftermarket rents. The optimal allocation of the shares aims at minimizing the sum of those rents. While the dispersion of shares increases aftermarket rents, favoring informed clients increases the cost of extracting their information. Indeed, an investor is more reluctant to misreport low interest if doing so precludes him from getting a share. But if he is certain to be served no matter what information he reveals, an investor who has a high appraisal may be tempted to report that he has a low appraisal to pay a lower offer price. Not surprisingly we find that the shares are concentrated in the hands of the informed clients when the aftermarket rents are high relative to informational rents. However, dispersion is optimal for a range of the parameters even when uninformed investors trade strategically. It is interesting to focus on how the solution varies depending on which type of investors (informed versus uninformed) can extract the aftermarket rents. When it is the uninformed investors, the optimal IPO goes along that of Benveniste and Spindt (1989) and Bennouri and Falconieri (2006). Priority is given to informed clients reporting high appraisals. If these are long-term holders then the potentially remaining shares are dispersed. The greater the aftermarket rents the narrower the range of parameters for which the underwriter relies on uninformed investors. When informed investors can extract both the aftermarket rents and the informational rents a new, particularly challenging, situation arises. The most interesting finding is that, unless clients reporting high appraisals can absorb the issue, information no longer comes first. Indeed for some values of the parameters, it is optimal to ration informed investors whether or not they report high interest. Provided aftermarket rents are relatively low, the dispersion of

1The informational rents refer to the surplus given to informed investors only, in exchange for their information.
the shares is optimal and priority is given to uninformed investors. This is so because these investors extract no rents for themselves. However they generate some liquidity trading in the aftermarket for the benefit of the informed clients. These aftermarket rents can be used to induce truthful revelations. The optimal IPO is complex but we can highlight the different trade-offs and cases for which dispersion is optimal.

The analysis of the costs and benefits of aftermarket trading helps understand different IPO practices either observed or tested empirically. The losses associated with a greater dispersion of the shares can explain why, in most IPOs, a large proportion of the bulk is typically sold to large institutional investors and why it is very difficult for retail investors to buy some shares. Our analysis supports Ritter and Welch (2002) and Loffler et al. (2005) suggesting that asymmetric information cannot account on its own for the amount of money left on the table. Indeed the aftermarket trading generates a new source of rents.

The fact that flipping by informed investors facilitates information revelation may also explain the findings in Aggarwal (2000) and Aggarwal (2002) stating that penalty bids are used selectively and that they are seldom assessed. Finally, Jenkinson and Jones (2009) and Goyal and Tam (2009) give evidence that the allocation of shares is biased in favor of long-term holders. Here, we show that informational rents increase as an investor pledges to be a long-term holder. Bringing together these results suggests that information acquisition from long term holders does not necessarily require a greater underpricing of the stock but rather a distortion in the allocation.

The trade-off between providing liquidity and underpricing is the focus of Booth and Chua (1996), Ellul and Pagano (2006) and very recently Busaba and Chang (2010). The first two papers argue that investors require less underpricing as the aftermarket liquidity increases. However while Ellul and Pagano (2006) proves that, as a result, the IPO proceeds increase with the aftermarket liquidity, Booth and Chua (1996) reaches the opposite conclusion. The latter paper shows that underpricing increases with dispersion in the presence of costly information acquisition.

Busaba and Chang (2010) is somehow closer to our analysis. They compare book building to fixed price offerings and provide evidence that, under both mechanisms, underpricing and liquidity are positively correlated. In their setting, informed investors who manage to retain their information can use it to extract aftermarket rents. More liquidity means greater aftermarket rents for informed investors. Thus, in book building as liquidity increases, these investors need more underpricing to reveal their information and thus sacrifice aftermarket rents. In fixed price there is no gathering of information during the IPO and it is now the uninformed investors who need a greater underpricing as liquidity increases to compensate them for aftermarket losses.

The sub-optimality of a uniform price rule is the focus of Chen and Wilhelm (2008). They find that when relevant information arrives in the aftermarket,
it is not optimal to sell shares at a uniform price. These authors show how a strategic allocation of the shares to institutional investors, together with price stabilization practices from underwriters in the aftermarket, enable to overcome the uniform pricing constraint so as to reach a more efficient outcome.

The controversial flipping has been the focus of a few papers. Krigman et al. (1999), Ellis (2006) and Aggarwal (2003) among others aim at identifying who are the flippers and how much of the issue is immediately sold. From these papers we learn that institutional investors do more flipping than retail investors and that some of the flipping is based on information that has not been revealed during the IPO. Boehmer and Fishe (2000) shows that the underwriter can gain from promoting flipping. Indeed, if the underwriter is also a market maker in the aftermarket and receives trading fees, he can create liquidity by under-pricing the issue and allocating shares to low valuation investors who flip the shares. We show that flipping can benefit the underwriter even when his objective function is the same as the issuer’s.

The paper unfolds as follows. The next section describes the model we use throughout the paper. In section 3 we derive the bids or willingness to pay of both informed and uninformed investors. In section 4 we characterize the optimal IPO by highlighting first the costs and benefits of aftermarket trading. Finally section 5 concludes.

2 Modeling the IPO

- The underwriter and the investors.

We consider that the underwriter’s objective is to maximize the expected proceeds of the IPO. While we acknowledge that agency problems may arise between the issuer and the investment bank or underwriter we adopt a normative approach and leave these aside. The investors attending the IPO are either informed (type I) or uninformed (type U).

The informed investors can be thought of as large institutional investors considered as elite clients by the underwriter. Their input is critical. As reported in Killian et al. (2001), the capital market desks (the management and the bankers) regularly consult high profile investors when setting an offer price. They are informed in the sense that they have an either high or low interest for the stock and, more importantly, their appraisal matters in that it is reflected in the value of the shares.

The uninformed investors are other investors who stand a chance of buying IPO shares but do not have access to private information during the road show. The underwriter knows if an investor is informed or not.

2Cornelli and Goldreich (2003) shows that the issue price is influenced by large bids submitted by investors who participate frequently to IPOs. Daniel (2002) tells how, during the Microsoft IPO, Goldman Sachs warned Gates about the disastrous implications of losing a few “high quality investors” as Gates was reluctant to lower the offer price.
A total of $Q$ shares must be allocated. In total there are $I$ risk-neutral investors interested in the issue. Of these, $n$ ($n \leq I$) are informed investors. To simplify we assume that each investor wishes to buy at most one share and that the issue is oversubscribed in the sense that there are enough informed investors to absorb the issue ($I > n \geq Q$). Let $\eta^i$ denote informed investor $i$’s interest for the stock ($i \in [0, n]$). Each $\eta^i$ is an i.i.d. realization of a random variable $\bar{\eta} \in \{-\eta, +\eta\}$ such that $\bar{\eta} = +\eta$ with probability $q \in [0, 1]$. We refer to state $k$, with $k \leq n$, as the situation where $k$ informed investors report high interest (i.e. $+\eta$). We assume that the value of a share in state $k$ is given by

$$V_k = v + \eta_k + \varepsilon,$$

\begin{equation}
(1)
\end{equation}

where $\eta_k = \eta(2k - n)$, where $v > 0$ is a commonly known variable and finally where $\varepsilon$ reflects the residual uncertainty. We assume that $\varepsilon$ is a random variable with $\varepsilon \in \{+\varepsilon, -\varepsilon\}$ and has zero expectation so that $\varepsilon = +\varepsilon$ with probability $\frac{1}{2}$. Notice that $E_{\varepsilon}(V_{k+1} - V_k) = 2\eta$, where $E_{\varepsilon}$ denotes the expectation over $\varepsilon$.

The aftermarket.

The aftermarket is modeled as a competitive dealership market as in Glosten and Milgrom (1985). With probability $\beta^t$ ($t = I, U$) a type $t$ investor learns no additional information and flips his share if he faces liquidity needs. We assume that liquidity needs arise with probability $z^t$ (respectively $z^U$) for the informed (uninformed) investors. We have $z^t \in [0, 1]$ for $t = I, U$. With probability $(1 - \beta^t)$ a type $t$ investor learns the realized value of $\varepsilon$ and uses this information to trade his share.

The most important feature of the aftermarket is that we do not assume that the information possessed by informed investors during the IPO becomes public knowledge. Instead we assume that the information revealed by informed investors is aggregated in the offer price and correctly interpreted by all agents participating in the aftermarket. Although in equilibrium investors report their interest truthfully, this setting incorporates a strategic dimension absent from models assuming that the private information becomes public knowledge: informed investors affect the bid price when they misreport their interest.

Finally, although we do not allow for short selling we comment on the implications it may have before we conclude.

The timing.

First the IPO is announced and the informed investors learn their interest for the shares. Then, the underwriter sets an allocation and pricing rule for the
stock taking into account the information revealed by the informed investors. Finally the offer price and allocation become common knowledge and the aftermarket opens.

We solve for the optimal offer prices and allocations of shares considering that investors perfectly anticipate the outcome of the aftermarket.

3 The investors’ willingness to pay.

Once an offer price is set and an allocation decided, each investor about to receive a share has the possibility to step out of the deal if he feels the offer price is greater than what he expects the share to be worth. A share’s value depends on whether the investor sells it or keeps it. If he keeps the share then it will be worth \( V_k \) in state \( k \) which is given by (1). If he sells the share in the aftermarket then he receives the state contingent bid price which is denoted \( p^b_k \). Let \( \rho^I \) (respectively \( \rho^U \)) denote the probability with which an informed (uninformed) investor keeps the share. We have

\[
\rho^t = \beta^t \left( 1 - z^t \right) + (1 - \beta^t) \frac{1}{2} \quad \text{for } t = I, U,
\]

given that with probability \( \beta^t \) the investor does not learn the value of \( \tilde{e} \) and keeps the share unless he faces liquidity needs, and with probability \( (1 - \beta^t) \) the investor learns the value of \( \tilde{e} \) and keeps the share provided \( \tilde{e} = +\varepsilon \).

In state \( k \), the highest offer price a type \( t \) investor is willing to pay is given by

\[
b^t_k = (1 - \rho^t) p^b_k + \beta^t \left( 1 - z^t \right) (v + \eta_k) + (1 - \beta^t) \frac{1}{2} (v + \eta_k + \varepsilon). \tag{2}
\]

(Recall that the expected value of \( \tilde{e} \) is zero.) To complete the characterization of \( b^t_k \) we now calculate the state contingent competitive bid prices.

As in Ellul and Pagano (2006) we assume that in the aftermarket every transaction takes place between a single trader and a dealer. A single potential trader is selected at random by the dealer. The dealer presents her competitive quotes to the trader who then decides whether to trade or not. The dealer knows the IPO pricing and allocation rules as well as \( n \). She can infer the state of nature \( (k) \) as she observes the offer price. From the allocation rule she can infer the proportion of shares held by each type of investors. The bid price reflects the expected value of a share given all the information gathered.

Let \( x_k(s) \) denote the probability that an informed investor who revealed interest \( s \in \{+\eta, -\eta\} \) in state \( k \) gets a share. Let \( y_k \) denote the state contingent probability with which an uninformed investor gets a share. Full allocation of the shares implies that \( \forall n \) and \( \forall k \leq n \)

\[
k x_k(+\eta) + (n - k) x_k(-\eta) + (I - n) y_k = Q. \tag{3}
\]

\footnote{The competitive bid price is such that it is optimal for an investor accessing information in the aftermarket to keep the share when \( \tilde{e} = +\varepsilon \) and to sell it when \( \tilde{e} = -\varepsilon \).}
Given that each type of investor trades with probability \((1 - \rho^I)\), the probability of facing a sell order in state \(k\) is given by

\[
\theta_k = \alpha_k \left( 1 - \rho^U \right) + (1 - \alpha_k) \left( 1 - \rho^I \right),
\]

(4)

where \(\alpha_k = \frac{(I - n) y_k}{Q}\) is the proportion of (previously) uninformed investors who received some shares.

The competitive bid price, in state \(k\), is then given by

\[
p^b_k = \phi^L (v + \eta_k) + \left( 1 - \phi^L \right) (v + \eta_k - \varepsilon),
\]

where

\[
\phi^L = \frac{\alpha_k \beta^U z^U + (1 - \alpha_k) \beta^I z^I}{\theta_k}
\]

(5)

is the proportion of sell orders motivated by liquidity needs.

The expression for the state contingent bid price simplifies to

\[
p^b_k = v + \eta_k - \frac{\varepsilon}{2\theta_k} \left[ 1 - \alpha_k \beta^U - (1 - \alpha_k) \beta^I \right].
\]

(6)

Plugging the bid price above in (2), leads to

\[
\begin{aligned}
\begin{cases}
\quad b^I_k = v + \eta_k - \frac{\varepsilon \alpha_k}{2\theta_k} \Delta, \\
\quad b^U_k = v + \eta_k + \frac{\varepsilon (1 - \alpha_k)}{2\theta_k} \Delta.
\end{cases}
\end{aligned}
\]

(7)

where

\[
\Delta = \beta^I z^I \left( 1 - \beta^U \right) - \beta^U z^U \left( 1 - \beta^I \right).
\]

(8)

Although 16 possible scenarios may arise in the aftermarket, in only 2 of these will one type of investors make a profit at the expense of the other and the sign of \(\Delta\) determines who stands to gain. Aftermarket rents arise when both types of investors sell or flip their share and one type has liquidity needs while the other type knows that \(\varepsilon = -\varepsilon\). With probability \(\beta^I z^I \left( 1 - \beta^U \right) \frac{1}{2}\) uninformed investors make a profit, while the informed clients do so with probability \(\beta^U z^U \left( 1 - \beta^I \right) \frac{1}{2}\). We have \(\Delta \neq 0\) when the two scenarios are not equally likely to arise. Note that when \(\beta^I = \beta^U\), \(\Delta = 0 \iff z^I = z^U\), and when \(z^I = z^U\), \(\Delta = 0 \iff \beta^I = \beta^U\).

From (7) we see that the prospect of extracting aftermarket rents leads investors to bid more aggressively. In that respect, we would expect institutional investors who are experienced enough to learn information in the aftermarket and who are large enough not to face substantial liquidity needs to be the highest bidders.
4 The optimal IPO.

The optimal state contingent offer prices and allocations maximize the expected proceeds:

\[ W = E_k p_k^0 Q, \]

where \( E_k \) is the expectation over all states of nature. The solution must satisfy the following constraints:

- Full allocation of the shares as given by (3) above.

- The allocations and offer prices must lead informed investors to reveal their interest truthfully. When reporting high or low interest an investor does not know which state of the world he is in. He reports \( s' \in \{+\eta, -\eta\} \) maximizing his expected revenue where expectation is taken over all possible states of the world given his own information. Let \( U(s, s') \) denotes an informed investor's expected revenue when his true interest is \( s \) while he reports \( s' \). We must have

\[
\begin{cases} 
U (+\eta, +\eta) \geq U (+\eta, -\eta), \\
U (-\eta, +\eta) \geq U (+\eta, -\eta).
\end{cases}
\]

(10)

As explained below, investors with low interest cannot gain from misreporting their interest so that the first constraint is the only one restricting the underwriter’s choices.

- Finally the investors have the possibility to opt out when proposed the offer price. In every state of the world, the offer price is bounded above by the lowest winning bid:

\[
(k x_k (+\eta) + (n - k) x_k (-\eta)) (b^I_k - p_k^0) \geq 0,
\]

\[
y_k (b^U_k - p_k^0) \geq 0,
\]

(11)

for all \( k \). When the shares are dispersed among both types of investors we must have

\[
p_k^0 \leq \min \{ b^I_k, b^U_k \}.
\]

(12)

Benveniste and Spindt (1989) describes the optimal allocation and offer prices when investors are long-term holders. They show that the allocation of shares must favour investors reporting high interest. The offer price matches the investors’ bids for all \( k < Q \) while the issue is underpriced when \( k \geq Q \). This result is very intuitive. The allocation minimizes informational rents and the state contingent offer prices guarantee that investors reporting low interest get no rents. Bennouri and Falconieri (2006) shows how the allocation is modified when introducing uninformed investors. It shows that allocating some shares to uninformed bidders can raise the revenue by decreasing the informational rents.

How does aftermarket trading affect the allocation and pricing? To answer that we highlight first the costs and benefits of aftermarket trading.
4.1 The costs and benefits of aftermarket trading.

If flipping IPO shares is typically frowned upon or perceived as detrimental for the issuing company, why is it tolerated? Indeed according to Aggarwal (2000) and Aggarwal (2002) penalty bids, which aim at discouraging flipping, are used selectively and are seldom assessed. Our first result sheds some light on why allowing selected investors to flip is beneficial.

**Lemma 1:** Allowing informed investors to sell their shares in the aftermarket reduces the cost of extracting information during the IPO stage.

(Proof: see Appendix.)

The incentive compatibility constraint for investors with high interest can be rewritten as:

\[ U(\eta, \eta) \geq U(-\eta, -\eta) + 2\eta \rho^i \sum_{k=0}^{n-1} \pi'_k x_k(-\eta). \tag{14} \]

Investors may be tempted to downgrade their interest to get a discount. However a lower offer price also means a lower bid price. Thus, the more likely an informed investor is to sell his share in the aftermarket, the lower his incentive to misreport his interest. This intuitive argument is reflected in (14) where the second term on the right hand side denotes the so-called informational rents. These are increasing as \( \rho^i \), the probability of keeping the share, rises.\(^7\)

Two observations follow from expression (14). The first relates to the allocation of shares. The other supports findings in the empirical literature according to which long-term holders seem to receive more shares.

**Corollary 1:** Favoring informed clients and securing their access to IPO shares increases the cost of extracting their information.

Indeed, as (14) shows, informational rents increase with \( x_k(-\eta) \). As we have seen the incentive constraint aims at eliciting information from investors who have high interest given that the others have no incentive to lie. The threat of not receiving any share when reporting low interest facilitates truthful reports.

**Corollary 2:** Among informed investors, long term holders require higher rents than flippers to reveal their information truthfully.

Jenkinson and Jones (2009) and Goyal and Tam (2009) state that the allocation of shares is biased in favor of long-term holders. Bringing together these empirical results and our result suggests that information acquisition from long term holders does not necessarily require a greater underpricing of the stock but rather a distortion in the allocation.

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\(^6\)As we shall see, the offer price increases the more informed investors report high interest.

\(^7\)An informed investor with low interest cannot benefit from misreporting his interest. Indeed, if he pretends to have high interest he risks paying an offer price greater than what he believes a share is truly worth. This would, in turn, increase the bid price but not sufficiently so that the investor would gain from such a lie. Thus, despite aftermarket trading, an investor with low interest has no incentive to report high interest.
Among the few papers examining the benefits of flipping in the aftermarket, Boehmer and Fishe (2000) shows that the underwriter gains from promoting flipping when he is also a market maker in the aftermarket where he receives trading fees. That would suggest that his objective function differs from the issuer’s. We show that allowing flipping does not necessarily stem from a conflict of interest between the issuer and the underwriter.

Having understood the benefit of flipping we now turn to its cost.

**Lemma 2:** After market trading decreases the IPO proceeds when (i), (ii) and (iii) hold simultaneously:

(i) the value of the share is subject to some residual uncertainty ($\varepsilon \neq 0$),
(ii) the shares are dispersed among informed and uninformed investors,
(iii) some investors are able to make a profit selling their share in the aftermarket ($\Delta \neq 0$).

**Proof:** Straightforward from the expression of the difference of the bids.

When one type of investors can make a profit in the aftermarket at the expense of the other there is a positive discrepancy between the bids given by

$$|b_i^U - b_i^I| = \frac{\varepsilon}{2\theta_k} |\Delta|.$$  \hspace{1cm} (15)

As we know investors who extract aftermarket rents bid higher. However, when both types of investors are served the offer price must match the lowest bid. It cannot be set so as to extract the revenue gathered in the aftermarket.

**Corollary 3:** Due to the uniform pricing rule the dispersion of IPO shares among heterogeneous investors potentially decreases the proceeds.

This result explains why, for most IPOs, a large proportion of the bulk is sold to large institutional investors and it is very difficult for retail investors to access shares. Selling to investors who exhibit greater heterogeneity leads to more money being left on the table. Along this line of research it is worth signaling Chen and Wilhelm (2008) who also point to the sub-optimality of the uniform pricing rule when some informational asymmetry between investors remains. To reduce the cost induced from the uniform pricing constraint, they show that it is optimal for the investment bank to allocate shares to investors whose aftermarket trading they can influence.

### 4.2 The optimal allocation of shares

We can rewrite the proceeds as

$$W = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - (I - n)\hat{U}$$

$$-nqU(+\eta, +\eta) - n (1 - q) U(-\eta, -\eta).$$  \hspace{1cm} (16)
where \( \hat{U} \) denote the rents to uninformed investors. To gather the whole surplus (expressed as the first term of the right hand side) the underwriter must try and eliminate both the informational rents and the rents gathered in the secondary market.

**Lemma 3:** When there are enough uninformed investors to absorb the issue \((I - n \geq Q)\), it is possible to extract the whole surplus by allocating all the shares to uninformed investors.

**Proof:** Uninformed investors do not require any informational rents and aftermarket rents only arise under dispersion. While mathematically correct this allocation of shares has two important drawbacks. First it is such that informed investors receive no shares whether or not they express high interest. In that respect this solution cannot be implemented when information acquisition and/or participation is costly.\(^8\) Second, one should not deduce from Lemma 3 that the number of informed investors should be minimized. Indeed, increasing the number of informed investors can generate a positive externality. For \( q > \frac{1}{2} \) the first term of \( W_n \) increases as more informed investors report high interest.

We consider now the cases where there are not enough uninformed investors to exhaust the issue: \( Q > I - n \). The different states of nature can be separated as follows:

1. Strong demand for the issue: informed investors reporting \(+\eta\) are numerous enough to exhaust the issue \((k \geq Q)\).
2. Moderate demand for the issue: informed investors reporting \(+\eta\) cannot exhaust issue but the investment bank need not rely on those reporting low interest \((Q - (I - n) \leq k < Q)\).
3. Weak demand for the issue: a large number of informed investors report low interest so that the investment bank must rely on them to exhaust the issue \((k < Q - (I - n))\).

Although the characteristics of the optimal IPO depend on whether \( \Delta > 0 \), \( \Delta = 0 \) or \( \Delta < 0 \), proposition 1 highlights the only feature that is immune to changes in the sign of \( \Delta \).

**Proposition 1:** When there is a strong demand for the issue, the stock is entirely allocated to informed investors reporting high interest \( (i.e. +\eta) \).

**Proof:** The proceeds decrease as part of the shares are allocated either to uninformed investors or to investors reporting low interest. Indeed, the informational rents are proportional to \( x_k (-\eta) \) and the aftermarket rents are proportional to \( y_k \) as they result from dispersing the shares among heterogeneous investors. Setting \( x_k (+\eta) = \min \left\{ 1, \frac{Q}{k} \right\} \) for \( k \geq Q \) leads to \( y_k = x_k (-\eta) = 0 \) which minimizes both types of rents.\(^8\)

\(^8\)Costly information acquisition is a complex topic in itself and is analyzed in details in Sherman (2000) and Sherman and Titman (2002).
Proposition 2 highlights the main features of the optimal allocation when $\Delta > 0$. A detailed characterization can be found in the appendix.

**Proposition 2: Optimal IPO when uninformed investors capture the aftermarket rents ($\Delta > 0$).** When the demand for the issue is either moderate or weak, priority is given to investors reporting high interest (i.e. $x_k(+\eta) = 1$). The optimal allocation of the remaining shares trades-off the informational rents with the rents gathered by uninformed investors in the aftermarket. The optimal offer prices match the informed investors’ bids for all $k \leq Q-1$ (i.e. $p^*_k = b^*_k$). For $k \geq Q$, the offer prices are such that the incentive constraint binds:

$$\sum_{k=Q}^{n} \pi_k \left[ b^*_k - p^*_k \right] = 2\eta \rho \left[ \frac{q}{1-q} \right] \sum_{k=0}^{Q-1} \pi_k \left( \frac{Q-k}{Q} \right).$$

(17)

Allocating shares to uninformed investors lowers the bids of informed investors (and thus the offer price) and generates aftermarket rents that the underwriter cannot extract due to the uniform price rule. Thus, intuition suggests that uninformed investors should be discarded. There is however a benefit ensuing from dispersion: it reduces the number of shares available to investors reporting low interest which in turn decreases the level of informational rents. When informational rents are high, that is when informed investors are long term holders (when $\rho'$ is high), it is optimal to disperse the shares among heterogeneous investors.

Figures 1 to 4 represent the optimal allocation as a function of $\rho'$ and $\rho''$. Technically, $\rho' \in \left[ \frac{1}{2} (1-\beta'), \frac{1}{2} (1+\beta') \right]$ but it is easier to represent the solution over the wider space $[0, 1]^2$.

From the figures, it is clear that, for any given $\rho'$, $y_k$ increases with $\rho''$ which corresponds with the intuition given above. It is also clear that uninformed investors are more likely to be discarded as $\Delta$ increases.

Finally, the optimal allocation is sensitive to the variable $\chi = \frac{(1-q)\varepsilon}{q\eta}$. This variable measures the relative costs of the two adverse selection problems. While the aftermarket rents are proportional to $\varepsilon$, informational rents are proportional to $\eta \frac{q}{1-q}$ as shown in (17). Given that uninformed investors capture the aftermarket rents when $\Delta > 0$, setting $y_k = 0$ is optimal for a wider range of parameters as $\chi$ increases.

**Lemma 4:** When $\Delta = 0$, shares are allocated according to the following priority order:

1- informed investors reporting high interest,
2- uninformed investors,
3- informed investors reporting low interest.

That is $x_k (+\eta) = \min \left\{ 1, \frac{Q}{k} \right\}$, $y_k = \min \left\{ 1, \frac{Q-k}{I-n} \right\}$ and $x_k (-\eta) = \max \left\{ 0, \frac{Q-k-(I-n)}{n-k} \right\}$. The optimal offer prices match the informed

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9The figures can be found in a Section following the Conclusion.
investors bids for all \( k \leq Q - 1 \) (i.e. \( p^*_{k} = b^I_{k} \)). For \( k \geq Q \), the offer prices are such that the incentive constraint binds as in (17).

We now consider the situation where informed investors stand to gain in the aftermarket. When \( \Delta < 0 \) the uninformed investors' bids are lower than the informed investors' bids. This has an important implication: when the underwriter allocates part of the stock to the uninformed investors the offer price must match their bid. Thus informed investors get a positive premium given by (15). This implies that the underwriter has an additional option to reward those reporting high interest. While he can still allocate shares to those reporting low interest, he may now discard those investors and allocate shares to uninformed investors.\(^{10}\)

Given that Proposition 1 still holds, the problem, for the underwriter is to establish in which states \( k < Q \) it is optimal to allow informed investors to get aftermarket rents so as to minimize the overall rents extracted by all informed investors without compromising the incentive compatibility constraint. While, we are not able to fully characterize the optimal IPO as precisely as we did for the case \( \Delta > 0 \), we can establish the following results.

**Lemma 5:** When the demand for the stock is either weak or moderate, the optimal value for \( y_k \) is either 0 or 1. Among informed investors priority is given to those reporting high interest.

The main point in Lemma 5 is that conditional on receiving shares, uninformed investors are given priority. Uninformed investors are not in a position to gather any rents because, conditional on being served, the offer price matches their willingness to pay. Informed investors can potentially extract both: the informational and aftermarket rents. As shown in Appendix, when demand is either weak or moderate, the optimal offer prices are such that the state contingent aftermarket rents resulting from serving a proportion \( \alpha_k \) of uninformed investors are given by

\[
C(\alpha_k) = (1 - \alpha_k) \frac{\varepsilon(-\Delta)}{2\theta_k}.
\]

Note that \( C(\cdot) \) equals the proportion of informed investors acquiring shares times the margin each of these gains. While the margin may increase or decrease as a function of \( \alpha_k \), the overall rents are always decreasing in \( \alpha_k \). Thus, the best alternative to setting \( y_k = 0 \) which eliminates aftermarket rents, is to set \( y_k = 1 \) which minimizes these rents.

Given Lemma 5, we then approach the design of the optimal IPO as follows. We first characterize the optimal allocation and offer prices conditional on discarding uninformed investors. This case is straightforward and follows directly from Benveniste and Spindt (1989). Priority is given to informed investors reporting high interest. The optimal offer prices match the informed investors bids for all \( k \leq Q - 1 \) (i.e. \( p^*_{k} = b^I_{k} \)). For \( k \geq Q \), the offer prices are such that the incentive constraint binds as in (17).

\(^{10}\)Unfortunately it also implies that the objective function is now discontinuous depending on whether \( y_k = 0 \) or \( y_k > 0 \). This complicates the analysis substantially.
The question is whether we can improve upon such an allocation by setting \( y_k = 1 \) for some \( k \).

**Proposition 3:** Informed investors capture the aftermarket rents \((\Delta < 0)\). For some \((\rho^I, \rho^U)\) such that \( \Delta < 0 \), giving priority to uninformed investors for some \( k \leq Q - 1 \) is optimal.

While setting \( y_k = 1 \) generates some aftermarket rents, it decreases informational rents by diminishing the number of shares left to informed investors reporting low interest. Consider the following allocation rule whereby uninformed investors are served only when demand is moderate:

\[
\begin{array}{c|c|c}
\text{Weak demand} & \text{Moderate demand} & \text{Strong demand} \\
\hline
y_k = 0 & y_k = 1 & y_k = 0 \\
\frac{k}{n-k}x_k(+) = 1 & \frac{Q-k}{Q-k}x_k(+) = Q-\frac{(1-n)}{k} & x_k(+) = Q \\
x_k(-) = 0 & x_k(-) = 0 & x_k(-) = 0 \\
\end{array}
\]

Table 1: Proposed allocation rule.

The advantage of this allocation is that informed investors reporting low interest do not extract any rents. The drawback is that \( x_k(-\eta) \) is high when demand is weak. Given this allocation rule, the question is whether there exist some offer prices that satisfy (10) and (13) which would lead to higher proceeds than those achieved when uninformed investors are discarded.

The graph below shows the region for which one allocation prevails over the other.

*Insert graph*

In region 3 \((b^I_k - b^U_k)\) is very large. When setting \( y_k = 1 \) the underwriter leaves substantial aftermarket rents to investors reporting high interest. Even when setting the offer price equal to the investors’ bids in all other states, the underwriter cannot get compensation for such losses and is better off setting \( y_k = 0 \) for all \( k \).

In region 2, when setting \( y_k = 1 \), the underwriter still leaves large aftermarket rents to investors reporting high interest. However the underwriter is now better-off using the allocation depicted in table 1 as he is able to recoup part of the losses by setting \( p^I_k = b^I_k \) for \( k \geq Q \). Interestingly, the incentive compatibility constraint does not bind in that region.

In region 1, the aftermarket rents, when available, are low. The optimal offer prices implementing the allocation in table 1 are such that the incentive constraint binds and they generate higher proceeds that those implementing \( y_k = 0 \) for all \( k \).

**Corollary 4:** When they can be compensated for their information by extracting aftermarket rents, it is not always optimal to give priority to informed investors reporting high interest.

This result is in sharp contrast with Benveniste and Spindt (1989).

In general the optimal allocation and offer prices depend on \( q \) and on the number informed investors.
5 Conclusion

In this paper we analyze an optimal IPO design incorporating aftermarket trading which allows us to consider distinct adverse selection problems one affecting the pre-market and the other affecting the aftermarket. We show that the money left on the table comprises the usual informational rents as well as a aftermarket rents associated with some outstanding uncertainty prevalent in the aftermarket. The informational rents are necessary to induce truthful revelation from informed investors with high interest. The aftermarket rents only arise when the shares are spread among heterogeneous investors. They are, in fact, a shortcoming of the uniform pricing rule which applies to all IPOs.

While the informational rents are captured by informed investors reporting high interest, the aftermarket rents can, a priori, be captured by either informed investors or uninformed investors depending on which type of investors is more likely to have access to superior information. In general, the underwriter maximizes the proceeds trading-off both rents. Not surprisingly, we find that it is optimal not to spread the shares among heterogeneous investors when the residual uncertainty is large. This is so because the aftermarket rents are proportional to the residual uncertainty. Perhaps more surprisingly, we find that informed investors reporting high interest for the issue are not systematically given priority. When informed investors are able to extract both, the informational and aftermarket rents, it can be optimal to give priority to uninformed investors. Informational rents are high when the informed investors interest can affect the value of the share substantially. When this is so, the underwriter prefers to compensate those reporting high interest by allowing them to extract aftermarket rents. However, he balances the cost this strategy triggers by limiting the number of shares available to these investors.

One repercussion of the analysis is that allowing informed investors to flip their share in the aftermarket has some benefits. First it reduces informational rents. An investor who is likely to flip a share cares about the bid price and is reluctant to affect it adversely. Second, aftermarket rents can also be a reward for reporting information during the IPO.

Theoretically, our paper could be extended by considering that investors have different flipping strategies. This can be accommodated in a model that assumes that both, the interest and the probability of flipping share, are private information. Alternatively an interesting model could be closer in spirit to Laffont and Tirole (1986) whereby the value of an investor combines his private information (adverse selection) and his intention to hold the asset over a long-term or short-term period (moral hazard). Another interesting line of research consists in analyzing how much information an investor chooses to reveal during the IPO knowing that he may be in a position to keep an informational advantage in the aftermarket if he keeps part of the information.
6 Figures

Figure 1: Optimal $y_k$ for $k < Q$ and $\frac{1}{4} \left( \beta^I - \beta^U \right) > 1$.

Figure 2: Optimal $y_k$ for $k < Q$ and $0 < \frac{1}{4} \left( \beta^I - \beta^U \right) < 1$. 
Figure 3: Optimal $y_k$ for $k < Q$ and $\beta^I = \beta^U$.

Figure 4: Optimal $y_k$ for $k < Q$ and $\beta^I < \beta^U$. 
Figure 5: Regions 1, 2 and 3 when $\beta^I > \beta^U$.

Figure 6: Regions 1, 2 and 3 when $\beta^I < \beta^U$. 
7 Appendix

- Important notation.

The unconditional probability of state \( k \) is given by

\[
\pi_k = \left( \frac{n}{k} \right) q^k (1 - q)^{n-k}.
\]

From the point of view of an informed investor with low interest state \( k \) occurs with probability

\[
\pi_k' = \left( \frac{n-1}{k} \right) q^k (1 - q)^{n-1-k}.
\]

Finally, state \( k \) occurs with probability \( \pi_{k-1}' \) for an informed investor with high interest.

The allocation rule is given by \( \{x_k(+\eta), x_k(-\eta), y_k\} \).

We then use the following notation:

\[
\alpha_k = \left( \frac{I - n}{Q} \right) y_k,
\]

\[
\rho^t = \beta^t (1 - z) + (1 - \beta^t) \frac{1}{2} \text{ for } t = I, U,
\]

\[
\theta_k = \alpha_k (1 - \rho^U) + (1 - \alpha_k) (1 - \rho^I),
\]

\[
\Delta = \left( 1 - \beta^U \right) \left( 1 - \rho^I \right) - \left( 1 - \beta^I \right) \left( 1 - \rho^U \right),
\]

and finally

\[
\chi = \frac{(1 - q) \varepsilon}{q \eta}.
\]

- Proof of Lemma 1.

The incentive compatibility constraint states that investors with high interest must reveal it truthfully. An informed investor honesty reporting high interest receives on expectation:

\[
U(+\eta, +\eta) = \sum_{k=0}^{n-1} \pi_k' x_{k+1}(+\eta) \left[ b_{k+1}' - p_{k+1}' \right],
\]

(18)

while an informed investor honestly reporting low interest gets

\[
U(-\eta, -\eta) = \sum_{k=0}^{n-1} \pi_k' x_k(-\eta) \left[ b_k'^t - p_k'^t \right].
\]

(19)
The expected profit that would result from reporting low interest while true interest is high is given by

\[ U(\eta, -\eta) = \sum_{k=0}^{n-1} \pi_k x_k(-\eta) \left[ (1 - \rho') p_k^0 + \rho'(v + \eta_{k+1}) + \frac{1}{2} \left( 1 - \beta^f \right) \varepsilon - p_k^0 \right]. \]

(20)

Misreporting interest changes the offer price which also affects the bid price on the aftermarket. Adding and subtracting \( \eta_k \rho' \) in the above leads to

\[ U(+\eta, -\eta) = U(-\eta, -\eta) + 2\eta \rho' \sum_{k=0}^{n-1} \pi'_k x_k(-\eta). \]

Thus, the IC constraint is such that

\[ U(+\eta, +\eta) \geq U(-\eta, -\eta) + 2\eta \rho' \sum_{k=0}^{n-1} \pi'_k x_k(-\eta). \]

(21)

This concludes the proof for Lemma 1.

- Rewriting the proceeds.

Given the full allocation of the shares, we can rewrite the objective of the seller as

\[ \text{Max} \sum_{k=0}^{n} \pi_k [k x_k(+\eta) + (n - k) x_k(-\eta) + (I - n) y_k] p_k^0. \]

Note that \( x_n(-\eta) = 0 \) and that \( x_0(+\eta) = 0 \) and finally that

\[ \pi_k = \frac{n(1 - q)}{n - k} \pi'_k \text{ and } \pi_{k+1} = \frac{nq}{k + 1} \pi_k'. \]

(22)

Given the above, as well as (7), we have

\[ \sum_{k=0}^{n} \pi_k x_k(+\eta) p_k^0 = nq \sum_{k=0}^{n-1} \pi'_k b_{k+1}^l x_{k+1} (+\eta) - nq U(+\eta, +\eta), \]

and

\[ \sum_{k=0}^{n} \pi_k(x - k) x_k(-\eta) p_k^0 = n (1 - q) \sum_{k=0}^{n-1} \pi'_k b_{k+1}^l x_{k+1} (-\eta) - n (1 - q) U(-\eta, -\eta). \]

We can rewrite the objective function as

\[ W = nq \sum_{k=0}^{n-1} \pi'_k b_{k+1}^l x_{k+1} (+\eta) + n (1 - q) \sum_{k=0}^{n-1} \pi'_k b_{k+1}^l x_{k} (-\eta) \]

(23)

\[ + \sum_{k=0}^{n} \pi_k(I - n) y_k p_k^0 - n (1 - q) U(-\eta, -\eta) - nq U(+\eta, +\eta). \]
Note that
\[ nq \sum_{k=0}^{n-1} \pi'_k b'_{k+1} x_{k+1} (+\eta) = \sum_{k=0}^{n} k \pi_k x_k (+\eta) b'_k, \]
and
\[ n(1-q) \sum_{k=0}^{n-1} \pi'_k b'_k x_k (-\eta) = \sum_{k=0}^{n} (n-k) \pi_k x_k (-\eta) b'_k, \]
and finally that
\[ \sum_{k=0}^{n} \pi_k(I-n)y_k p'_k = \sum_{k=0}^{n} \pi_k(I-n)y_k (p'_k - b'_k) + \sum_{k=0}^{n} \pi_k(I-n)y_k b'_k \]
Substituting the above in (23) we obtain
\[
W = \sum_{k=0}^{n} \pi_k(I-n)y_k (p'_k - b'_k) - n(1-q) \sum_{k=0}^{n} \pi_k x_k (-\eta) - nqU(+\eta, +\eta) \\
+ \sum_{k=0}^{n} \pi_k b'_k \left[kx_k (+\eta) + (n-k) x_k (-\eta) + (I-n)y_k \right].
\]
Given that
\[ kx_k (+\eta) + (n-k) x_k (-\eta) + (I-n)y_k = Q, \]
we have
\[
W = Q \sum_{k=0}^{n} \pi_k b'_k + \sum_{k=0}^{n} \pi_k(I-n)y_k (p'_k - b'_k) \\
- n(1-q) \sum_{k=0}^{n} \pi_k x_k (-\eta) - nqU(+\eta, +\eta).
\]
Considering the definition of \( b'_k \) \((t = U, I)\) and noting that
\[ b'_k - b'_k = \frac{\varepsilon \Delta}{2\hat{\theta}_k}, \]
we can rewrite the above as
\[
W = Q \sum_{k=0}^{n} \pi_k(v + \eta_k) - \sum_{k=0}^{n} \pi_k(I-n)y_k (b'_k - p'_k) - nqU(+\eta, +\eta) (24) \\
- n(1-q) \sum_{k=0}^{n} \pi_k x_k (-\eta) - nqU(+\eta, +\eta).
\]
• **Optimal mechanism when** \( \Delta > 0 \).

Note that we have \( \Delta > 0 \iff b'_k > b'_k \). Given that there are not enough uninformed investors to absorb the issue, the only relevant ex-post incentive
constraint is given by \( p_k^0 \leq b_k^I \). Consider (24) above. Given a binding incentive constraint it can be re-written as

\[
W = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - \sum_{k=0}^{n} \pi_k (I - n) y_k (b_k^I - p_k^0) - nq \left[ 2\eta \rho' \sum_{k=0}^{n-1} \pi'_k x_k (-\eta) \right] - nU (-\eta, -\eta).
\]

Given the above, it is obviously optimal to set \( x_k (+\eta) \) as high as possible as doing so reduces all negative terms. Therefore, for any \( k \geq Q \) all the shares go to investors reporting high interest and for any such \( k \) we have \( x_k (-\eta) = 0 \) and \( y_k = 0 \). It is also obviously optimal to set \( p_k^0 = b_k^I \) for \( k = 0, \ldots, Q - 1 \) while for \( k \geq Q \) the offer prices \( p_k^0 \) are such that the IC constraint binds. Incorporating the offer prices in the proceeds function leads to

\[
W = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - 2nq \eta \rho' \sum_{k=0}^{Q-1} \pi'_k x_k (-\eta) - (I - n) \sum_{k=0}^{Q-1} \pi_k y_k \frac{\varepsilon \Delta}{2\theta_k}.
\]

Finally, using the full allocation constraint given by

\[(n - k) x_k (-\eta) + (I - n) y_k = Q - k, \]

we can rewrite the objective function as:

\[
W = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - 2 \eta \frac{q}{1 - q} \rho' \sum_{k=0}^{Q-1} \pi_k (Q - k) + (I - n) \sum_{k=0}^{Q-1} \pi_k f(y_k),
\]

where

\[
f(y_k) = y_k \left[ 2\eta \frac{q}{1 - q} \rho' - \frac{\varepsilon \Delta}{2\theta_k} \right].
\]

The optimal \( y_k \) for \( k = 0, \ldots, Q - 1 \) is the value that maximizes \( f(y_k) \).

Lemma A1: The state contingent objective function \( f(y_k) \) is concave for \( \rho' > \rho^I \), linear for \( \rho' = \rho^I \) and convex for \( \rho^I > \rho' \).

Proof: We have

\[
\frac{df}{dy_k} = 2\eta \frac{q}{1 - q} \rho' - \frac{\varepsilon (1 - \rho^I) \Delta}{2(\theta_k)^2},
\]

and

\[
\frac{d^2f}{dy_k^2} = \frac{\varepsilon (1 - \rho^I)}{2Q (\theta_k)^3} (I - n) (\rho^I - \rho') \Delta.
\]

\footnote{Setting \( p_k^0 = b_k^I \) for \( k = 0, \ldots, Q - 1 \) also guarantees that \( U (-\eta, -\eta) = 0 \).}
Consider the following functions:

\[ \frac{df}{dy_k} \bigg|_{y_k=0} \geq 0 \Leftrightarrow F_0 (\rho^I, \rho^U) \geq 0, \]

where

\[ F_0 (\rho^I, \rho^U) = 4 \rho^I (1 - \rho^I) - \chi \Delta. \]

\[ \frac{df}{dy_k} \bigg|_{y_k=\min\{1, \frac{k}{n}\}} \geq 0 \Leftrightarrow F_1 (\rho^I, \rho^U) \geq 0, \]

where

\[ F_1 (\rho^I, \rho^U) = 4 \rho^I \left[ \theta_k |_{y_k=\min\{1, \frac{k}{n}\}} \right]^2 - \chi (1 - \rho^I) \Delta. \]

And finally, we have

\[ f \left( y_k = \min \left\{ 1, \frac{Q-k}{T-n} \right\} \right) \geq f(0) \Leftrightarrow F_2 (\rho^I, \rho^U) \geq 0 \]

where

\[ F_2 (\rho^I, \rho^U) = 4 \rho^I \left[ \theta_k |_{y_k=\min\{1, \frac{k}{n}\}} \right] - \chi \Delta. \]

The 3 equations

\[ F_s = 0 \quad \text{with} \quad s = 0, 1, 2, \]

classify an implicit function \( \rho^U (\rho^I) \). Indeed, for any given \( \rho^I \) there is an unique \( \rho^U \) that satisfies the above equation for \( s = 0, 1, 2 \). (We skip this proof as it trivial.\(^{12}\)) To evaluate the optimal allocation we represent these implicit functions in the \([0, 1]^2\) space with \( \rho^I \) on the horizontal axis.\(^{13}\) All 3 functions intersect at \( \rho^I = \rho^U = 1 \), \( \rho^I = \rho^U = \frac{1}{4} \left( \beta^I - \beta^U \right) \) and \( \rho^I, \rho^U = \left( 0, \frac{\beta^U - \beta^I}{1-\beta^I} \right) \).

The point(s) \( (1, 1) \) and \( \left( \frac{1}{4} \left( \beta^I - \beta^U \right), \frac{1}{4} \left( \beta^I - \beta^U \right) \right) \) are the unique solution(s) on the 45 degree line.\(^{14}\) Finally when \( \rho^I = 0 \), all 3 equations hold when \( \Delta = 0 \) so that all three functions intersect at \( \left( 0, \frac{\beta^U - \beta^I}{1-\beta^I} \right) \).

- If \( \beta^I - \beta^U \geq 0 \) and \( \frac{1}{4} \left( \beta^I - \beta^U \right) < 1 \) the 3 functions are below the 45 degree line for all \( \rho^I \in \left[ 0, \frac{1}{4} \left( \beta^I - \beta^U \right) \right] \), they then cut the 45 degree line at \( \rho^I = \rho^U = \frac{1}{4} \left( \beta^I - \beta^U \right) \) and are above the 45 degree line for all \( \rho^I \in \left[ \frac{1}{4} \left( \beta^I - \beta^U \right), 1 \right] \).

- If \( \beta^I - \beta^U \geq 0 \) and \( \frac{1}{4} \left( \beta^I - \beta^U \right) > 1 \) the 3 functions are below the 45 degree line for all \( \rho^I \in [0, 1] \), they then cut the 45 degree line at \( \rho^I = \rho^U = 1 \).

\(^{12}\)The equations \( F_0 = 0 \) and \( F_2 = 0 \) lead to an implicit concave function \( \rho^U (\rho^I) \). For \( F_1 \), considering \( y = (1 - \rho^U) \), setting \( F_1 = 0 \) leads to a polynomial of the form \( ay^2 + by + c = 0 \).

\(^{13}\)Clearly \( \rho^I \in \left( \frac{1}{2} (1 - \beta^I), \frac{1}{2} (1 + \beta^I) \right) \) as \( z \) varies from 0 to 1. However, it is easier to do the analysis for the wider space \([0, 1]^2\).

\(^{14}\)This would be a single point if \( \frac{1}{4} \left( \beta^I - \beta^U \right) = 1 \).
- If $\beta^I - \beta^U < 0$ the 3 functions are above the 45 degree line for all $\rho^I \in [0, 1]$, they then cut the 45 degree line at $\rho^I = \rho^U = 1$.

**Lemma A2:** For any $(\rho^I, \rho^U) \in [0, 1] \times [0, 1]$ the following holds:

\[
F_0 (\rho^I, \rho^U) > F_2 (\rho^I, \rho^U) > F_1 (\rho^I, \rho^U) \text{ for any } \rho^U > \rho^I
\]

\[
F_1 (\rho^I, \rho^U) > F_2 (\rho^I, \rho^U) > F_0 (\rho^I, \rho^U) \text{ for any } \rho^I > \rho^U.
\]

Proof:

- Comparing $F_0$ and $F_1$.
  Consider all $(\rho^I, \rho^U)$ such that $F_0 (\rho^I, \rho^U) = 0$:
  \[4\rho^I (1 - \rho^I) = \chi \Delta.\]
  Evaluating $F_1 (\rho^I, \rho^U)$ at such points we get
  \[
  \text{sign of } F_1 (\rho^I, \rho^U) = \text{sign of } (\rho^I - \rho^U).
  \]

- Comparing $F_0$ and $F_2$.
  Consider all $(\rho^I, \rho^U)$ such that $F_0 (\rho^I, \rho^U) = 0$:
  \[4\rho^I (1 - \rho^I) = \chi \Delta.\]
  Evaluating $F_2 (\rho^I, \rho^U)$ at such points we get
  \[
  \text{sign of } F_2 (\rho^I, \rho^U) = \text{sign of } (\rho^I - \rho^U).
  \]

- Comparing $F_1$ and $F_2$.
  Consider all $(\rho^I, \rho^U)$ such that $F_2 (\rho^I, \rho^U) = 0$:
  \[4\rho^I [\theta_k |_{y_k = \min \{1, \frac{q}{1-q}\}}] = \chi \Delta.\]
  Evaluating $F_1 (\rho^I, \rho^U)$ at such points we get
  \[
  \text{sign of } F_1 (\rho^I, \rho^U) = \text{sign of } (\rho^I - \rho^U).
  \]

This concludes the description of the implicit functions. Considering what each equation means together with the result stated in Lemma A1 fully characterizes the optimal allocation. Note that, where defined, the interior solution solves

\[
\frac{df}{dy_k} = 2\eta \frac{q}{1-q} \rho^I - \frac{\varepsilon (1 - \rho^I) \Delta}{2 (\theta_k)^2} = 0.
\]

- **Proof of Lemma 4.**
When $\Delta = 0$, the state contingent objective function given by (28) is linear and increasing in $y_k$. Therefore, once investors reporting $+\eta$ have been served it is optimal to set $y_k = \min \left\{ 1, \frac{Q-k}{I-n} \right\}$.

- **Optimal mechanism when $\Delta < 0$.**

Let us split the states of nature into the 3 categories mentioned in the text: $K_L = [0, Q - (I - n) - 1]$, $K_M = [Q - (I - n), Q - 1]$ and $K_H = [Q, n]$.

The proceeds function is given by:

$$W = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - (I - n) \sum_{k=0}^{n} \pi_k y_k (b_k^U - p_k^o) - nqU (+\eta, +\eta)$$

$$-n(1-q)U(-\eta, -\eta).$$

Since $b_k^U < b_k^I$ we have $p_k^o = b_k^U$ when $y_k \neq 0$. Thus, the second term, denoting the uninformed investors’ rents, is now equal to zero.

Considering the expressions for $U (+\eta, +\eta)$ and $U (-\eta, -\eta)$ we can re-write the maximization problem as

$$\max Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - Q \sum_{k=0}^{n} \pi_k [1 - \alpha_k] (b_k^I - p_k^o),$$

subject to (21). Without any calculations, we can establish (i) and (ii):

(i) It is obvious that for all $k \in K_H$, it is optimal to allocate all shares to the investors reporting $+\eta$ so that $y_k = x_k(-\eta) = 0$ for all $k \geq Q$, which minimizes rents. The offer price can be as high as $b_k^I$ provided the incentive constraint is satisfied.

(ii) Since informed investors with low interest need no rents to report their information truthfully, the optimal offer price for any $k \leq Q - 1$ is such that

$$p_k^o = \begin{cases} b_k^I & \text{when } \alpha_k = 0, \\ b_k^U & \text{when } \alpha_k \neq 0. \end{cases}$$

**Lemma A3:** For any $k \leq Q - 1$, the optimal value for $y_k$ is either 0 or 1. When $y_k = 0$, it is optimal to set $x_k (+\eta) = 1$ and $x_k(-\eta) = \frac{Q-k}{n-k}$ when $y_k = 1$ it is optimal to allocate the remaining shares such that $x_k (+\eta) = \frac{Q-(I-n)}{k}$ and $x_k(-\eta) = 0$ for $k \in K_M$ and $x_k (+\eta) = 1$ and $x_k(-\eta) = \frac{Q-k-(I-n)}{n-k}$ for $k \in K_L$.

Proof: Given (i) and (ii), for any $k \leq Q - 1$ the second term of $W$ term is positive if and only if $\alpha_k \neq 0$. For any $k$, the function

$$C(\alpha_k) = \frac{(1 - \alpha_k) \varepsilon(-\Delta)}{26_k}$$

is always decreasing in $\alpha_k$. Thus conditional on setting $y_k > 0$, it is optimal to set $y_k = 1$. Finally, whether or not uninformed investors have been served, it

$^{15}$For $\rho^I < \rho^U$ the function is concave and the derivative is negative as $\alpha_k \to 0$. 

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is obvious that it is optimal to allocate the remaining shares to those reporting 
+\eta first. Doing so reduces the informational rents by reducing \( x_k(-\eta) \). When 
y_k = 1 giving priority to those reporting +\eta is also a way to compensate them 
at no additional cost for revealing their information truthfully.

When \( y_k = 0 \) for all \( k \), the solution is such that \( x_k(+) = \min \left\{ 1, \frac{Q}{n} \right\} \) and 
\( x_k(-\eta) = \max \left\{ 0, \frac{Q-k}{n-\epsilon} \right\} \). The incentive compatibility constraint binds and we have

\[
W|_{y_k=0} = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - 2(nq) \eta \rho \sum_{k=0}^{Q-1} \pi'_k \left( \frac{Q - k}{n - k} \right). \tag{31}
\]

Let us now consider the possibility that \( y_k = 1 \) for some \( k < Q \). Given (30) 
and the fact that \( x_k(+) = \frac{Q}{n} \) for \( k \in K_H \), we have

\[
W = Q \sum_{k=0}^{n} \pi_k (v + \eta_k) - G_1,
\]

where

\[
G_1 = Q \sum_{k=Q}^{n} \pi_k (b_k^* - p_k^*) - \sum_{k \leq Q-1 | y_k=1} \pi_k |Q - (I - n)| \left( \frac{\varepsilon \Delta}{2 \theta_k|y_k=1} \right). \tag{32}
\]

Note that, due to the fact that \( p_k^* \leq b_k^* \) for all \( k \), the solution be such that

\[
G_1 \geq c_0 \left( y_{k|k \in K_L}, y_{k|k \in K_M} \right), \tag{33}
\]

where

\[
c_0 \left( y_{k|k \in K_L}, y_{k|k \in K_M} \right) = \left( -\frac{\Delta \varepsilon}{2 \theta_k|y_k=1} \right) \sum_{k \leq Q-1 | y_k=1} \pi_k |Q - (I - n)|.
\]

When setting \( x_k(+) = \frac{Q}{n} \) for \( k \geq Q \), given (30) and the full allocation con-
straint where \( y_k = 1 \) for some \( k < Q \), the incentive compatibility constraint may 
be written as follows:

\[
G_1 \geq c_1 \left( y_{k|k \in K_L}, y_{k|k \in K_M} \right), \tag{34}
\]

where

\[
c_1 \left( y_{k|k \in K_L}, y_{k|k \in K_M} \right) = 2(nq) \eta \rho \sum_{k=0}^{Q-1} \pi'_k x_k (-\eta) 
- \sum_{k \leq Q-1 | y_k=1} \left( -\frac{\varepsilon \Delta}{2 \theta_k|y_k=1} \right) \pi_k \frac{(n - k) x_k (-\eta)}{1 - q}.
\]
Finally, setting \( y_k = 1 \) for some \( k < Q \) will improve upon \( y_k = 0 \) for all \( k \) provided

\[
G_1 \leq 2(nq) \eta \rho^t \sum_{k=0}^{Q-1} \pi'_k \left( \frac{Q-k}{n-k} \right) c_2,
\]

(35)

Therefore an improvement can be implemented if (33), (34) and (35) define a non-empty set.

Consider the allocation depicted in table 1. It can be implemented and improves upon \( y_k = 0 \) for all \( k \) provided

\[
\{ G_1 \geq \max \{ c_0(0,1), c_1(0,1) \} , \ G_1 \leq c_2 \}.
\]

Note that \( c_1(0,1) < c_2 \). Moreover we have \( c_1(0,1) \geq c_0(0,1) \) if and only if

\[
H_1(\rho^t, \rho^U) \geq 0
\]

where

\[
H_1(\rho^t, \rho^U) = 4\left( \theta_k \mid y_k = 1 \right) nq \eta \rho^t \sum_{k \in K_L \cup K_M} \pi'_k \left( \frac{Q-k}{n-k} \right) + \varepsilon \Delta \sum_{k \in K_M} \pi_k [Q - (I - n)].
\]

Finally, we have \( c_0(0,1) \leq c_2 \) if and only if

\[
H_2(\rho^t, \rho^U) \geq 0
\]

where

\[
H_2(\rho^t, \rho^U) = 4\left( \theta_k \mid y_k = 1 \right) nq \eta \rho^t \sum_{k \in K_L \cup K_M} \pi'_k \left( \frac{Q-k}{n-k} \right) + \varepsilon \Delta \sum_{k \in K_M} \pi_k [Q - (I - n)].
\]

Note that for each \( \rho^t \) there is a unique \( \rho^U \) solving \( H_s = 0 \) for \( s = 1, 2 \). Moreover it is such that \( \Delta < 0 \). Note also that the points \( (\rho^t, \rho^U) = (1, 1) \) and \( (\rho^t, \rho^U) = \left( 0, \frac{\beta - \beta^t}{1 - \beta^t} \right) \) are such that \( H_s = 0 \) for \( s = 1, 2 \). Finally, the curve \( H_1 = 0 \) crosses the 45 degree line at \( \rho^t = \rho^U = \frac{\varepsilon(\beta - \beta^t)}{4nq \sum_{k \in K_M} \pi_k \left( \frac{\beta - \beta^t}{n-k} \right) \sum_{k \in K_M} \pi_k [Q - (I - n)]} \). And the curve \( H_2 = 0 \) crosses the 45 degree line at \( \rho^t = \rho^U = \frac{\varepsilon(\beta - \beta^t)}{4nq \sum_{k \leq Q-1} \pi_k \left( \frac{\beta - \beta^t}{n-k} \right) \sum_{k \leq Q-1} \pi_k [Q - (I - n)]} \).

It is clear that where \( \Delta < 0 \) we have \( H_2(\rho^t, \rho^U) > H_1(\rho^t, \rho^U) \). Therefore, for all \( (\rho^t, \rho^U) \) such that \( H_1(\rho^t, \rho^U) = 0 \), we have \( H_2(\rho^t, \rho^U) > 0 \).

\[ \text{\footnotesize The reasoning is the same as that in the previous appendix for the functions } F_s s = 0, 1, 2. \]
The graph below represents the two curves $H_1 (\rho^I, \rho^U) = 0$ and $H_2 (\rho^I, \rho^U) = 0$ and splits the space where $\Delta \leq 0$ in 3 regions.

In region 1, we have $c_0 (0, 1) < c_1 (0, 1) < c_2$. Thus there exists offer prices that implement $y_k = 1$ for $k \in K_M$ such that the allocation proposed in deviation 1 is incentive compatible and improves upon $y_k = 0$ for all $k$. These offer prices are optimal when they are such that the incentive constraint binds: $G_1 = c_1 (0, 1)$.

In region 2, we have $c_1 (0, 1) < c_0 (0, 1) < c_2$. Thus there exists offer prices that implement $y_k = 1$ for $k \in K_M$ such that the allocation proposed in deviation 1 is incentive compatible and improves upon $y_k = 0$ for all $k$. These offer prices are such that $p^0_k = b^I_k$ for $k \geq Q$. Indeed, in this case the informed investors reporting $+\eta$ gather so much rents in the aftermarket that the incentive constraint does not bind.

In region 3 we have $c_1 (0, 1) < c_2 < c_0 (0, 1)$. Thus there does not exist offer prices that would be accepted by informed investors in states $k \geq Q$ that can implement $y_k = 1$ for $k \in K_M$ such that the allocation proposed in deviation 1 is incentive compatible.

8 Bibliography


