The Effect of Endogenous Human Capital Accumulation on Optimal Taxation

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Abstract

This paper considers the impact of learning-by-doing on optimal tax policy in a life cycle model. Analytically, it shows two channels by which learning-by-doing can alter the optimal tax policy. First, learning-by-doing creates a motive for the government to use age-dependent labor income taxes. If the government cannot condition taxes on age, then a capital tax or progressive/regressive labor income tax can be used in order to mimic age-dependent taxes. Second, a progressive/regressive labor income tax is potentially more distortionary in a model with learning-by-doing since the distortion is propagated through the additional intertemporal link between current labor and future human capital. Quantitatively, I find that both of these channels are important. Including learning-by-doing causes a 4.7 percentage point increase in the optimal capital tax primarily due to the first channel. Moreover, adding learning-by-doing leads to a notably flatter optimal labor income tax due to the second channel. I find that when solving for the optimal tax policy in the learning-by-doing model, the welfare consequences of not accounting for endogenous human capital accumulation are equivalent to 4.1 percent of expected lifetime consumption.


Key Words: Optimal Taxation, Capital Taxation, Progressive Taxation, Human Capital.

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1 Introduction

Both consumption and labor tend to vary over an individuals’ lifetimes. In part, this variation is driven by fluctuations in age-specific human capital.\textsuperscript{1} Atkeson et al. (1999), Erosa and Gervais (2002), Garriga (2001), and Gervais (2012) determine that because of this variation in age-specific human capital, it is optimal to condition taxes on age.\textsuperscript{2} Moreover, they demonstrate that if age-dependent taxes are not allowed, then it is possible to mimic the optimal tax policy with either a non-flat labor income tax (i.e. a progressive or regressive tax) or a non-zero tax on capital. Conesa et al. (2009), Peterman (2013), and Gervais (2012) demonstrate quantitatively in a life cycle model that the inability to condition taxes on age can be a strong motive for a positive capital tax and a progressive/regressive tax on labor income. Taken as a whole, these results suggest that variation in human capital over the lifetime has implications for two fundamental questions in the optimal taxation literature.\textsuperscript{3} First, should capital be taxed? Second, should the income tax be progressive?

Despite the importance of age-specific human capital, previous studies examining these optimal tax questions tend to assume human capital is accumulated exogenously. By imposing this assumption of exogenous accumulation, these previous studies ignore any effect that endogenous human capital accumulation may have on the optimal tax policy. In this paper, I both analytically and quantitatively determine the effect on optimal tax policy of including endogenous age-specific human capital accumulation through learning-by-doing (LBD). With LBD, an agent determines his level of future age-specific human capital by choosing the hours he works in the current period. Previous research empirically supports that there is a relationship between current work and future productivity. For example, Topel (1990), Cossa et al. (1999), and Altuğ and Miller (1998) show that past hours worked and length of current job tenure impact current wages. Moreover, LBD is is commonly employed in quantitative life cycle models used to answer other macroeconomic and public finance questions (for example, see Hansen and İmrohoroğlu (2009), Imai and Keane (2004), and Chang et al. (2002)).\textsuperscript{4} However, previous research has not determined the effect of LBD on optimal tax

\textsuperscript{1} I define age-specific human capital as human capital that is accumulated after an agent begins working.

\textsuperscript{2} Atkeson et al. (1999) demonstrate a related result. They show conditions under which the optimal capital tax is zero if age-dependent taxes on labor income are allowed.

\textsuperscript{3} See Diamond and Saez (2011) and Mirrlees et al. (2010) for a discussion of the importance of these questions and a general summary of previous findings

\textsuperscript{4} An alternative form of endogenous human capital accumulation that is sometimes used is learning-or-doing (LOD). In LOD, which is also referred to as Ben Porath type skill accumulation or on-the-job training, an agent acquires human capital by spending time training in periods in which he is also working. This paper ignores this form of human capital accumulation because Muligan (1995) finds that once individuals start working they spend less than 7 percent of their time endowment in formal training. Therefore, this form of human capital accumulation is more relevant to pre-work skill formulation than age-specific human capital accumulation. Peterman (2014) finds that in a model without within cohort heterogeneity there are much small effects on the optimal capital tax when endogenous human capital is added with LOD. Alternatively, Jacobs and Bovenberg (2009) finds that this incorporating endogenous pre-work skill accumulation has similar effects on the optimal tax policy as LBD.
policy in a life cycle model. In this paper, I find, both analytically and quantitatively, that including LBD affects both the optimal progressivity of the labor income tax as well as the optimal level of capital taxation.

I begin by analytically demonstrating, using a parsimonious two-generation model, that there are two channels through which LBD can affect the optimal tax policy. First, adding LBD changes the relative incentives to work over an agent’s lifetime. In a model with exogenous skill accumulation, an agent’s primary incentive to work is his wage. In contrast, in a model with LBD, the benefits from working are an agent’s current wage as well as an increase in his future age-specific human capital. I refer to these benefits as the “wage benefit” and the “human capital benefit,” respectively. The importance of the human capital benefit decreases as an agent approaches retirement. Thus, adding LBD causes the agent to supply labor relatively less elastically early in his life compared with later in his life. Optimally, the social planner would tax income from the less elastically supplied labor of younger agents at a relatively higher rate than older agents. However, if the social planner cannot use age-dependent taxes, then a tax on capital or progressive/regressive labor income tax can be optimal in order to mimic the age-dependent taxes. This optimal tax policy is in contrast to Garriga (2001) which demonstrates that, in a specific set of models with exogenous human capital accumulation, it is not optimal to condition labor taxes on age. Therefore, in the Garriga (2001) model with exogenous age-specific human capital accumulation, the optimal tax on capital is zero regardless of whether age-dependent taxes are allowed. I refer to this first channel as the elasticity channel.

Second, I analytically demonstrate that including LBD enhances the distortions from a non-flat labor income tax. The distortion is enhanced in the LBD model due to the additional intertemporal link between current labor and future human capital. In both the exogenous and LBD model a progressive tax distorts an agent’s labor decisions because the marginal wage benefit declines as labor income increases. However, in the LBD model a progressive labor income tax leads to an additional distortion because the marginal human capital benefit also declines as future labor income increases. Therefore, on its own, this additional distortions will lead to a flatter optimal labor income tax policy.

Next, I quantitatively assess the impact of LBD on optimal tax policy in a more rigorous calibrated overlapping generations model (OLG) that includes a reduced form social security program, lifetime length uncertainty, idiosyncratic productivity shocks, and idiosyncratic shocks to labor productivity. To explore the effect of endogenous human capital accumulation, I solve for the optimal tax policies in two different cases – first in a model with no LBD (the exogenous model) and then again in a model with LBD (the LBD model). Starting with the exogenous model, I find that the optimal tax policy can be approximated

\[5\] A host of work demonstrates a similar set of results in a two generation model with a single cohort. Two examples of these works include Atkinson and Stiglitz (1976), and Deaton (1979).
by a 25.2 percent flat tax on capital income and a 33.5 percent tax on labor income with a deduction of $7,750. The optimal tax policy includes both a positive tax on capital and a progressive tax on labor income. Unlike the analytically tractable model, the positive optimal tax on capital arises in this exogenous model because I include many additional features in this more rigorous model.\footnote{See Peterman (2013) for an in depth discussion of motives for a positive capital tax in calibrated OLG model.} Moreover, the progressive tax in the more rigorous model is optimal in order to redistribute from high to low income agents and provide partial insurance against idiosyncratic productivity shocks.\footnote{For a discussion of these channels see Mirrlees (1971), Stiglitz (1982), Mirrlees (1974), and Varian (1980). See Conesa et al. (2009) and Conesa and Krueger (2006) for an in depth discussion of the quantitative effect on optimal taxation of these productivity shocks.}

In contrast to the exogenous model, I find that the optimal tax policy in the computational LBD model is a 29.9 percent tax on capital income and a 20.9 percent tax on labor income with no deduction. Thus, adding LBD has large quantitative implications on both tax questions. In particular, adding LBD raises the optimal capital tax by 4.7 percentage points and eliminates the progressivity in the optimal labor income tax. Through a series of computational experiments, I demonstrate that the elasticity channel is primarily responsible for the increase in the optimal capital tax when LBD is included. Moreover, adding LBD leads to a flatter optimal labor income tax due to the second channel. In particular, in the model with LBD, a progressive tax distorts labor decisions by discouraging human capital accumulation. In both models, the social planner is determining the optimal trade off between efficiency (flat taxes) and equity (progressive taxes). Interestingly, adding LBD fundamentally alters this trade off, resulting in a progressive labor income tax no longer being optimal.

Overall, I find that the welfare consequences of not accounting for the effects of LBD when determining the optimal tax policy are large. In particular, I find that implementing the optimal tax policy from the exogenous model – which includes a progressive labor tax and lower capital tax – as opposed to the actual optimal tax policy – which includes a flat tax on labor and larger tax on capital – results in an average welfare reduction equivalent to 4.1 percent of expected lifetime consumption. Therefore, this paper demonstrates that including endogenous age-specific human capital accumulation has quantitatively important effects on the optimal tax policy.

This paper contributes to the general class of literature that explores the optimal tax policy when the set of available tax instruments are restricted. Correia (1996), Armenter and Albanesi (2009), and Jones et al. (1997), demonstrate that certain tax instruments, that would otherwise not be optimal when the government has access to a complete set of instruments, may become optimal with the government’s set of tax instruments are restricted. This paper combines two related strands of the literature within this class of research.
that quantitatively determine the optimal capital tax and optimal progressivity of the income tax when the government is restricted from using age-dependent taxes.

The first strand simultaneously examines both tax questions in a calibrated life cycle model but includes human capital accumulation exogenously. Conesa et al. (2009), henceforth CKK, find in a life cycle model that similar to my exogenous model the optimal tax policy includes both a progressive labor income tax and a sizeable tax on capital. The authors find that one primary reason for the large optimal tax on capital is to mimic an age-dependent tax. However, the authors demonstrate that the primary reason that the progressive labor income tax is optimal is to provide ex-ante insurance for idiosyncratic shocks to labor productivity and not to mimic age-dependent taxes. In contrast, Gervais (2012) examines the optimal tax policy in a simplified version of the CKK model and finds that in some cases, even with a large tax on capital, a mild amount of progressivity in the labor income tax is optimal in order to mimic an age-dependent tax policy. However, since these other studies include human capital accumulation exogenously, they ignore any affects of endogenous human capital accumulation on the optimal capital tax or the optimal progressivity of the labor income tax. This paper both analytically and quantitatively assess these effects on optimal tax policy from endogenous human capital accumulation.

This paper is related to a second strand of the literature that incorporates LBD but only focus on the effect of LBD on one of the tax questions. For example, focusing on optimal capital taxation, Chen et al. (2010) finds that, in an infinitely lived agent model with labor search, including endogenous human capital accumulation causes the optimal capital tax to increase because a higher capital tax unravels the labor market frictions in their model. Since Chen et al. (2010) only examine the effect of endogenous human capital on the optimal capital tax in an infinitely lived agent model, they are unable to assess whether LBD affects the motive to use age-dependent taxes and also whether LBD affects the efficiency of a progressive labor income tax versus a tax on capital to mimic age-dependent taxes. Focusing on the effect of LBD on optimal progressivity, both Best and Kleven (2012) and Krause (2009) demonstrate that a flatter income tax, as

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8There is a strand of literature that examines these questions in an infinitely lived agent model as opposed to a life cycle model. See Diamond and Saez (2011) for a review of this literature.

9Peterman (2013) demonstrates that an additional motive for a positive tax on capital is that the government is unable to distinguish between accidental bequests and ordinary capital income. Further work, such as, Karabarbounis (2012) and Peterman (2012), demonstrate that incorporating endogenous fluctuations in labor supply on the extensive margin can enhance this motive for the government to use a capital tax to mimic age-dependent taxes on labor income. In contrast, Cespedes and Kuklik (2012) find that when a non-linear mapping between hours and wages exists then hours tend to become more persistent and the optimal capital tax fall significantly, however is still positive.

10The authors include endogenously human capital accumulation through both LBD and also training. The labor market frictions in Chen et al. (2010) cause a lower level of employment in their economy. A capital tax causes the wage discount to increase, thus causing firms to post more vacancies which in turn causes an increase in worker participation. A number of studies examine the optimal tax policy in an infinitely lived agent model with other forms of endogenous human capital accumulation. Examples include Jones et al. (1997), Judd (1999), and Reis (2007).
opposed to a progressive tax, is optimal in a two-generation model with LBD so as to not discourage human capital accumulation. However, Best and Kleven (2012) and Krause (2009) do not incorporate savings so they do not determine the effect of LBD on the optimal capital tax. Since both a tax on capital and a progressive/regressive labor income tax can be used to mimic an age-dependent tax policy, it is important to examine the effect of LBD on both questions simultaneously in a life cycle model. Moreover, quantitatively determining how much welfare is lost from ignoring LBD when solving for both the optimal capital tax and optimal progressivity of the labor income tax determines whether modeling human capital accumulation endogenously is important when examining optimal tax policy. Overall, this paper combines both strands of the literature and determines that there are large effects on both the optimal capital tax and progressive labor tax in a life cycle model when LBD is included.

This paper is organized as follows: Section 2 examines an analytically tractable version of the model to demonstrate the two channel by which LBD alters the optimal tax policy. Section 3 describes the full model and the competitive equilibrium used in the quantitative exercises. The calibration and functional forms are discussed in section 4. Section 5 describes the computational experiment, and section 6 presents the results. Section 7 tests the sensitivity of the results with respect to the shape of the labor supply profile, while section 8 concludes.

2 Analytical Model

In this section, I demonstrate that there are two potential channels by which adding LBD alters the optimal tax policy. First, I show that adding LBD causes the government to want to condition labor income taxes on age because it alters the relative labor supply elasticity over the lifetime. I show this result using a utility function that is homothetic and separable in both consumption and labor to demonstrates that LBD overturns the results from Garriga (2001) that under this type of utility function, with exogenous age-specific human capital accumulation, the government has no incentive to condition taxes on age. It is useful to determine if the government wants to use age-dependent taxes because Garriga (2001), Erosa and Gervais (2002), and Gervais (2012) show that if the government wants to condition taxes on age and cannot do so then generally a non-zero tax on capital or progressive/regressive labor income tax will be optimal in order to mimic this age-dependent tax. Second, I show that adding LBD enhances the distortions from a progressive tax.

I derive these analytical results in a tractable two-period version of the computational model. For tractability purposes, the features I abstract from include: retirement, population growth, idiosyncratic labor productivity shocks, and conditional survivability. Additionally, I assume that the marginal products of cap-
ital and labor are constant. This assumption permits me to focus on the life cycle elements of the model, in
that changes to the tax system do not affect the pre-tax wage or rate of return. Since the factor prices do not
vary, I suppress their time subscripts in this section. Also, because I exclude idiosyncratic labor productivity
shocks, there is no within cohort heterogeneity in the analytically tractable model. Therefore, without this
within cohort heterogeneity the social planner focuses only on efficiency and ignores the tradeoff between
equity and efficiency. All of these assumptions are relaxed in the computational model.

2.1 Exogenous Age-Specific Human Capital

2.1.1 General Set-up

I begin by examining the analytically tractable model with exogenous age-specific human capital as a bench-
mark to compare the LBD model. In the analytically tractable model, agents live with certainty for two
periods, and their preferences over consumption and labor are represented by

\[ U(c_{1,t}, h_{1,t}) + \beta U(c_{2,t+1}, h_{2,t+1}) \]  

(1)

where \( \beta \) is the discount rate, \( c_{j,t} \) is the consumption of an age \( j \) agent at time \( t \), and \( h_{j,t} \) is the percent of
the time endowment the agent works. Age-specific human capital is normalized to unity when the agent
is young. At age two, age-specific human capital is \( \varepsilon_2 \). The agent maximizes equation 1 with respect to
consumption and hours subject to the following constraints

\[ c_{1,t} + a_{1,t} = (1 - \tau_{h,1})h_{1,t}w \]  

(2)

and

\[ c_{2,t+1} = (1 + r(1 - \tau_k))a_{1,t} + (1 - \tau_{h,2})\varepsilon_2h_{2,t+1}w, \]  

(3)

where \( a_{1,t} \) is the amount young agents save, \( \tau_{h,j} \) is the tax rate on labor income for an agent of age \( j \), \( \tau_k \) is
the tax rate on capital income, \( w \) is the efficiency wage for labor services, and \( r \) is the rental rate on capital.
I begin by assuming that the tax rate on labor income is flat but can be conditioned on age. Moreover, I
assume that the tax rate on capital income is flat and cannot be conditioned on age.\(^{11}\) I combine equations 2

\(^{11}\)Agents only live for two periods in the analytically tractable model so they choose not to save when they are old. Therefore, in
this model restricting capital tax policy to not be age-dependent is not binding.
and 3 to form a joint intertemporal budget constraint:

\[ c_{1,t} + \frac{c_{2,t+1}}{1 + r(1 - \tau)} = w(1 - \tau_{h,1})h_{1,t} + \frac{w(1 - \tau_{h,2})\varepsilon_2 h_{2,t+1}}{1 + r(1 - \tau)}. \] (4)

The agent’s first order conditions are

\[ \frac{U_{h1}(t)}{U_{c1}(t)} = -w(1 - \tau_{h,1}), \] (5)

\[ \frac{U_{h2}(t + 1)}{U_{c2}(t + 1)} = -w\varepsilon_2(1 - \tau_{h,2}), \] (6)

and

\[ \frac{U_{c1}(t)}{U_{c2}(t + 1)} = \beta(1 + r(1 - \tau)), \] (7)

where \( U_{c1}(t) \equiv \frac{\partial U(c_{1,t}, h_{1,t})}{\partial c_{1,t}} \). Given a social welfare function, prices, and taxes, these first order conditions, combined with the intertemporal budget constraint, determine the optimal allocation of \((c_{1,t}, h_{1,t}, c_{2,t+1}, h_{2,t+1})\).

2.1.2 Primal Approach

I use the primal approach to determine the optimal tax policy which implies that the social planner maximizes directly over allocations.\(^{12}\) I use a social welfare function that maximizes the expected utility of a newborn and discounts future generations with social discount factor \(\theta\),

\[ [U(c_{2,0}, h_{2,0})/\theta] + \sum_{t=0}^{\infty} \theta^t [U(c_{1,t}, h_{1,t}) + \beta U(c_{2,t+1}, h_{2,t+1})]. \] (8)

The social planner maximizes this objective function with respect to two constraints: the implementability constraint and the resource constraint.\(^{13}\) The implementability constraint is the agent’s intertemporal budget constraint, with prices and taxes replaced by his first order conditions (equations 5, 6, and 7)

\[ c_{1,t}U_{c1}(t) + \beta c_{2,t+1}U_{c2}(t + 1) + h_{1,t}U_{h1}(t) + \beta h_{2,t+1}U_{h2}(t + 1) = 0. \] (9)

\(^{12}\)See Lucas and Stokey (1983) or Erosa and Gervais (2002) for a full description of the primal approach.

\(^{13}\)The government budget constraint is a third constraint. Due to Walras’ Law, I only need to include two of three constraints in the Lagrangian and leave out the government budget constraint.
Including this constraint ensures that any allocation the social planner chooses can be supported by a competitive equilibrium. The resource constraint is

\[ c_{1,t} + c_{2,t} + K_{t+1} - K_t + G_t = rK_t + w(h_{1,t} + h_{2,t} \varepsilon_2). \]  

As a benchmark utility function, I use \( U(c,h) = \frac{c^{1-\sigma_1}}{1-\sigma_1} - \chi \frac{(h^{1+\frac{1}{\sigma_2}})}{1+\frac{1}{\sigma_2}}. \) This utility function meets the conditions described in Garriga (2001) since it is homothetic and separable in consumption and labor. Incorporating this utility specification, the Lagrangian the social planner maximizes is

\[ \mathcal{L} = \frac{c^{1-\sigma_1}}{1-\sigma_1} - \chi \frac{h^{1+\frac{1}{\sigma_2}}}{1+\frac{1}{\sigma_2}} + \beta \frac{c^{1-\sigma_1}}{1-\sigma_1} - \chi \frac{h^{1+\frac{1}{\sigma_2}}}{1+\frac{1}{\sigma_2}} \]

\[ \quad - \rho_t(c_{1,t} + c_{2,t} + K_{t+1} - K_t + G_t - rK_t - w(h_{1,t} + h_{2,t} \varepsilon_2)) \]

\[ \quad - \rho_{t+1} \theta(c_{1,t+1} + c_{2,t+1} + K_{t+2} - K_{t+1} + G_{t+1} - rK_{t+1} - w(h_{1,t+1} + h_{2,t+1} \varepsilon_2)) \]

\[ \quad + \lambda_t \left( \frac{c^{1-\sigma_1}}{1-\sigma_1} + \beta \frac{c^{1-\sigma_1}}{1-\sigma_1} - \chi h^{1+\frac{1}{\sigma_2}} - \beta \chi h^{1+\frac{1}{\sigma_2}} \right) \]

where \( \rho \) is the Lagrange multiplier on the resource constraint and \( \lambda \) is the Lagrange multiplier on the implementability constraint.

### 2.1.3 Optimal Tax Policy

The formulation of the social planner’s problem and the first order conditions for the exogenous model can be found in appendix A.1. Combining the social planner’s first order conditions generates the following expression for optimal labor income taxes:

\[ \frac{1 - \tau_{h,2}}{1 - \tau_{h,1}} = \frac{1 + \lambda_t(1 + \frac{1}{\sigma_2})}{1 + \lambda_t(1 + \frac{1}{\sigma_2})} = 1. \]  

Equation 12 demonstrates the Garriga (2001) result, that with this type of utility function the social planner has no incentive to condition labor income taxes on age when age-specific human capital is exogenous.\(^{14}\)

Utilizing the first order condition from the Lagrangian with respect to capital and consumption leads to the following equation:

\[ \left( \frac{c_{1,t}}{c_{2,t+1}} \right)^{-\sigma_1} = \beta(1 + r). \]  

\(^{14}\)This result is specific to this utility function. See Garriga (2001) for further details.
Applying the benchmark utility function to equation 7 provides the following relationship:

\[
\left( \frac{c_{1,t}}{c_{2,t+1}} \right)^{-\sigma_i} = \beta(1 + r(1 - \tau_k)).
\] (14)

Equations 13 and 14 demonstrate that in order for the household to choose the optimal allocation indicated by the primal approach, the capital tax must equal zero.\(^\text{15}\)

2.1.4 Tax on Capital Mimics Age-Dependent Tax on Labor

Next, I demonstrate why the desire to condition labor taxes on age may affect the optimal capital tax by constructing the intertemporal Euler equation (derived from 5, 6, and 7):

\[
\varepsilon_2 \frac{U_{h1}(t)}{U_{h2}(t+1)} = \beta(1 + r(1 - \tau_k)) \frac{1 - \tau_{h1}}{1 - \tau_{h2}}.
\] (16)

Equation 16 demonstrates that if the social planner wants to create a wedge on the marginal rate of substitution by varying the labor income tax rate by age, then \(\tau_k\) is an alternative option. A positive (negative) capital tax induces a wedge on the marginal rate of substitution that is similar to a relatively larger tax on young (old) labor income.

2.1.5 Effect of Progressive/Regressive Tax in Exogenous Model

Gervais (2012) demonstrates in a quantitative model that under certain assumption the social planner may use a progressive/regressive labor income tax to mimic an age-dependent tax on labor income instead of a capital tax. In this section, I demonstrate why such a tax policy can mimic an age-dependent tax in the exogenous model and discuss the relative efficiency of a progressive/regressive labor income tax versus a tax on capital.\(^\text{16}\)

In this subsection, I assume that the average tax rate on labor income is no longer a function of age, instead it is a function of labor income \(T(h_{1,\text{w}} \varepsilon_i)\). This change in the tax function leads to a change in

\(^\text{15}\)Regardless of whether the government can condition labor income taxes on age, in this model the social planner does not want to tax capital because there is no desire to mimic an age-dependent tax on labor income. When the government cannot condition labor income taxes on age then the Lagrangian includes an additional constraint:

\[
\varepsilon_2 \frac{U_{h1}(t)}{U_{c1}(t)} = \frac{U_{h2}(t+1)}{U_{c2}(t+1)}.
\] (15)

However, in the analytically tractable model with exogenous human capital accumulation, this constraint is not binding and thus the Lagrange multiplier equals zero.

\(^\text{16}\)When allowing the government to use a progressive/regressive labor income tax the same primal approach does not yield an analytical solution because labor choices affect the average labor tax rate. Therefore, prices cannot be removed from the intertemporal budget constraint using the first order conditions in order to create the implementability constraint.
agent’s constraints (equations 17 and 18)

\[ c_{1,t} + a_{1,t} = (1 - T(h_{1,t}w))h_{1,t}w \tag{17} \]

and

\[ c_{2,t+1} = (1 + r(1 - \tau_k))a_{1,t} + (1 - T(\epsilon h_{2,t+1}w))\epsilon h_{2,t+1}w, \tag{18} \]

where \( T \) is the average tax rate on labor income which is assumed to be strictly increasing and concave in labor income. The agent’s first order conditions with this new tax function are

\[ \frac{U_{h1}(t)}{U_{c1}(t)} = -w\left(1 - T(wh_{1,t}) - h_{1,t}wT_{h1}(t)\right), \tag{19} \]

\[ \frac{U_{h2}(t+1)}{U_{c2}(t+1)} = -w\epsilon_{2}\left(1 - T(wh_{2,t}\epsilon_{2}) - h_{2,t}wT_{h2}(t+1)\right), \tag{20} \]

and

\[ \frac{U_{c1}(t)}{U_{c2}(t+1)} = \beta(1 + r(1 - \tau_k)), \tag{21} \]

where \( T_{h1}(t) \equiv \frac{\partial T(h_{1,t}w)}{\partial h_{1,t}w} \), or the marginal tax rate. The first order conditions with respect to labor (equations 19 and 20) have an addition term compared to the first order conditions in the model with flat labor taxes (equations 5 and 6). The additional term arises because with a progressive/regressive tax if an agent changes the hours they work then their average labor tax rate also changes.

The intertemporal Euler equation in this new setting is

\[ \epsilon_{2}\frac{U_{h1}(t)}{U_{h2}(t+1)} = \beta(1 + r(1 - \tau_k))\left(\frac{1 - T(wh_{1,t}) - h_{1,t}wT_{h1}(t)}{1 - T(wh_{2,t}\epsilon_{2}) - h_{2,t}wT_{h2}(t+1)}\right). \tag{22} \]

If the social planner wants to condition taxes on age and create a wedge in the marginal rate of substitution then he can either tax capital or use a progressive/regressive labor income tax. In particular, if labor income increases over an agent’s lifetime then a regressive labor income tax, or a tax on capital, would create a similar wedge as an age-dependent system that taxed young labor income at a higher rate.

Although both a progressive/regressive tax on labor income or a non-zero tax on capital can create a wedge on the marginal rate of substitution, there are several reasons why a tax on capital may be more desirable, especially in a more rigorous model. First, if the social planner wants the implicit labor income tax to monotonically decrease with age, a positive tax on capital may be ideal since it mimics this monotonously decreasing tax by age. In contrast, a progressive tax implicitly taxes labor income at a higher rate at ages
when an agent earns more. Therefore, if labor income is not monotonically increasing or decreasing over a working agent’s life then there is no way for a progressive/regressive tax policy to mimic a monotonically decreasing age-dependent tax policy. For example, if labor income peaks in the middle of an agent’s working lifetime then although a regressive labor income tax will implicitly tax young labor income at a higher rate it will also tax older labor income at a high rate.

The second reason an tax on capital may be preferable to a progressive/regressive labor income tax, is a capital tax imposes a wedge on the marginal rate of substitution that is independent of the agent’s labor choice. In contrast, the size of the wedge from a progressive/regressive labor income tax will depend on the amount of labor income. In a less parsimonious model that includes within cohort heterogeneity, agents of the same age may have different labor income, making it even more difficult for the social planner to use a progressive/regressive labor income tax to mimic an age-dependent tax. In contrast, the wedge from a capital income tax will only be a function of age and not labor supply so it will be the same for all agents of the same age regardless if there is within cohort heterogeneity with regards to labor income.\(^{17}\)

### 2.2 Learning-by-Doing

#### 2.2.1 Including LBD Creates Motive for Age-Dependent Taxes on Labor Income

Next, I introduce LBD into the exogenous model with flat labor income taxes. Similar to the exogenous model, I normalize age-specific human capital for young agents to one. Age-specific human capital for an old agent is determined by the function \(s_2(h_{1,t})\) which is an increasing and concave function with respect to hours worked when young. In this model agents maximize the same utility function subject to

\[
c_{1,t} + a_{1,t} = (1 - \tau_{h,1})h_{1,t}w
\]  

and

\[
c_{2,t+1} = (1 + r(1 - \tau_{h}))a_{1,t} + (1 - \tau_{h,2})s_2(h_{1,t})h_{2,t+1}w.
\]

The agent’s first order conditions are given by

\[
\frac{U_{h1}(t)}{U_{c1}(t)} = -[w(1 - \tau_{h,1}) + \beta U_{c2}(t + 1)w(1 - \tau_{h,2})h_{2,t+1}s_1(t + 1)],
\]

\[
\frac{U_{h2}(t + 1)}{U_{c2}(t + 1)} = -ws_2(h_{1,t})(1 - \tau_{h,2}),
\]

\(^{17}\)The wedge from the capital tax is only present if the agent holds some savings.
and
\[
\frac{U_{c1}(t)}{U_{c2}(t+1)} = \beta(1+r(1-\tau_k)).
\] (27)

The first order conditions with respect to \(h_2\) and \(a_1\) are similar in the LBD (equations 26 and 27) and exogenous models (equations 6 and 7). However, the first order condition with respect to \(h_1\) is different in the two models (equations 25 and 5) because working has the additional human capital benefit in the LBD model. This human capital benefit also alters the implementability constraint. Suppressing the arguments of the skills function, the implementability constraint in the LBD model is
\[
c_1, t U_{c1}(t) + \beta c_{2,t+1} U_{c2}(t+1) + h_1, t U_{h1}(t) - \frac{\beta h_1, t U_{h2}(t+1) h_2 s_1(t+1)}{s_2} + \beta h_{2,t+1} U_{h2}(t+1) = 0,
\] (28)

where \(s_1(t+1)\) represents the partial derivative of the skill function for an older agent with respect to hours worked when young.

The formulation for the social planner’s problem and the resulting first order conditions (utilizing the benchmark utility function) are in appendix A.2. Combining the first order conditions from the social planner’s problem and suppressing the time arguments yields the following ratio for optimal labor income taxes,
\[
\frac{1-\tau_{h,1}}{1-\tau_{h,2}} = \frac{\left[1 + h_2, t+1 s_2\right] \left[1 + \lambda(1 + h_2, t+1 s_2)(1 + \frac{1}{s_2})\right]}{1 + \lambda(1 + \frac{1}{s_2}) - \beta h_2, t+1 s_2 \frac{1}{\tau_2} \left[\frac{\lambda}{h_2, t+1 s_2} \left(1 + \lambda(1 + \frac{h_2, t+1 s_2}{s_2})\right)(1 + \frac{1}{s_2}) - h_2 \left(\frac{\lambda}{h_2, t+1 s_2}\right)^2 \frac{\lambda}{h_2, t+1 s_2} - \frac{\lambda h_2, t+1 s_2}{s_2}\right] - h_2, t+1 s_2}. \] (29)

Equation 29 demonstrates that generally in the LBD model the social planner has an incentive to condition labor income taxes on age. This result is in contrast to the exogenous model, in which the social planner has no incentive to condition labor income taxes on age (see equation 12).

In order to get a sense of which agent’s labor income the social planner might want to tax at a relatively higher rate, I examining the intertemporal Euler equation (determined by combining equations 25, 26 and 27):
\[
s_2(h_{1,t}) \frac{U_{h1}(t)}{U_{h2}(t+1)} = \beta(1+r(1-\tau_k)) \frac{1-\tau_{h,1}}{1-\tau_{h,2}} + \beta h_{2,t+1} s_1(t+1).
\] (30)

Including LBD causes the intertemporal Euler equation to have an extra term on the right hand side that is positive (see equation 16 and equation 30). Therefore, holding all else equal and setting \(\varepsilon_2 = s_2\), the tax on young labor income would need to be relatively higher in order to induce the same wedge on the marginal rate of substitution in the LBD model.

Examining the Frisch elasticities in the exogenous and LBD models, provides the intuition why adding
LBD may increase the optimal relative tax on young labor income. Since the functional forms of these elasticities extends to a model where agents live for more than two periods, I denote an agent’s age with $i$. In the exogenous model, the Frisch elasticity simplifies to $\Xi_{\text{exog}} = \sigma^2$. The Frisch elasticity in the LBD model is, $\Xi_{\text{LBD}} = \sigma^2 \frac{1}{1 - \frac{\beta h_{t+1} w_{t+1} \sigma^2}{h_{t+1} w_{t+1} \sigma^2}}$. In the LBD model, the extra terms in $\Xi_{\text{LBD}}$ increase the size of the denominator, thus holding hours and consumption constant between the two models, $\Xi_{\text{exog}} > \Xi_{\text{LBD}}$. Intuitively, the inclusion of the human capital benefit makes workers less responsive to a one-period change in wages since the wage benefit is only part of their total compensation for working in the LBD model. Moreover, the human capital benefit does not have a constant effect on an agent’s Frisch elasticity over his lifetime. The relative importance of the human capital benefit decreases over an agent’s lifetime because he has fewer periods to use his higher human capital as he ages. Therefore, adding LBD causes a young agent to supply labor relatively less elastically than an older agent. This shift in relative elasticities creates an incentive for the social planner to tax the labor income of younger agents at a relatively higher rate. I use the term “elasticity channel” to describe the effect on optimal tax policy caused by a change in the Frisch elasticity from including endogenous human capital.

If the government cannot condition labor income taxes on age, then either a capital tax or progressive/regressive labor income tax can be used to mimic such a tax policy. In particular, a larger tax on capital will mimic such an age-dependent tax policy. Moreover, if labor income is rising as an agent ages then a regressive labor income tax can be used to mimic such an age-dependent tax policy. Therefore, adding LBD alters the optimal tax policy.

### 2.2.2 LBD Enhances Distortions from Progress/Regressive Tax

This section compares the effectiveness of a progressive/regressive labor income tax and a capital tax at mimicking an age-dependent tax policy in the LBD model. Assuming the government uses a progressive/regressive labor income tax in the LBD model, the agents first order conditions, suppressing the arguments of the skill accumulation function, with respect to labor and capital are

$$\frac{U_{h_1}(t)}{U_{c_1}(t)} = -w \left( 1 - T(h_{1}, w) - h_{1} T_{h_1}(t) w + \beta \frac{U_{c_2}(t + 1)}{U_{c_1}(t)} h_{2,t+1} s_{h_1} \left( 1 - T(h_{2,t+1} w_{t+2}) - s_{2} T_{c_2}(t + 1) s_{h_1} (t + 1) w_{2,t+1} \right) \right), \quad (31)$$

---

18 This is the Frisch elasticity with respect to a temporary increase in the wage. Therefore, one must distinguish between $w_{t}$ and $w_{t+1}$.

19 Alternative intuition for this result can be demonstrated in the commodity tax framework of Corlett and Hague (1953). In their static framework, the social planner wants to tax leisure. However, if they cannot directly tax leisure, the social planner will tax commodities that are more complementary to leisure at a higher rate. Viewing this simple two generation model in that framework, adding LBD raises the relative opportunity cost of leisure when agents are young so young labor is less of a substitute (more of a complement) with leisure. This change leads the social planner to want to increase the tax on young labor. Moreover, if the social planner cannot use age-dependent taxes then they can increase the tax on capital to implicitly tax consumption from the old at a relatively higher rate since LBD makes consumption and leisure more complementary for the older agents than the younger agents.
Comparing the first order conditions in this model with LBD and progressive/regressive taxes (equations 34, 32 and 33) versus the first order conditions in the exogenous model with flat age-dependent taxes (equations 5, 6, and 7), the addition of LBD and the progressive/regressive tax changes the first order conditions with respect to labor. An agent’s first order conditions are altered for two reasons. First, LBD adds an intertemporal link between young labor and productivity when an agent is old. Second, the progressive/regressive labor tax implies that if agents work more their average tax rate will increase. Moreover, there is an interaction between these two effects. Focusing on the first order condition with respect to young labor, I classify the source of the additional terms that an agent incorporates when deciding to work,

\[ -w \left( 1 - T(h_{1,t}) - h_{1,t}T(h_{1,t}) + \beta \frac{U_{c2}(t + 1)}{U_{c1}(t)} \left( 1 - T(h_{2,t+1} + ws_2) - h_{2,t+1}T(h_{2,t+1}) \right) \right). \]  

The additional term from the interaction implies that the distortions from a progressive/regressive tax are enhanced in the LBD model because of the additional intertemporal link between current labor and future human capital. In both the exogenous and LBD models a progressive tax distorts an agent’s labor decisions because the marginal wage benefit declines as labor income increases. However, in the LBD model, a progressive tax policy will also reduce an agent’s incentives to work since the progressive tax implies that the marginal human capital benefit declines as future labor income increases. Therefore, adding LBD implies that the distortions from a progressive/regressive labor income tax are enhanced.

Like the exogenous model, the social planner can use a progressive/regressive tax or capital tax to mimic an age-dependent labor income tax in the LBD model. However, similar to the exogenous model, a capital tax may be more effective at mimicking an age-dependent tax policy since it is unaffected by the shape of the labor income profile and within cohort heterogeneity. Moreover, since the additional intertemporal link in the LBD model enhances the distortions from a progressive/regressive tax, LBD may generally make the social planner less willing to use a non-flat labor income tax as opposed to a capital tax to mimic age-dependent taxes.
3 Computational Model

Next, I determine the quantitative effect of adding LBD on optimal tax policy in a more rigorous version of the model that includes other channel that affect the optimal capital tax and progressivity of the labor income tax. It is important to include these channels since LBD may interact with these channels. One notable channel arises from the inclusion of within cohort heterogeneity which causes that the social planner to consider not only efficiency but weigh the tradeoff between efficiency and equity. In particular, the social planner can use a progressive labor income tax to redistribute and provide insurance against labor income risk. I solve for the optimal tax policy in separate versions of the computational model with exogenous human capital accumulation and LBD. The exogenous model is adapted from CKK; however I use a different utility function which is homothetic and separable so that the elasticity is not a function of hours worked in the exogenous model. This utility function is attractive because the constant Frisch labor supply elasticity in the exogenous model allows me to isolate the effects of the elasticity channel on the optimal tax policy in the LBD model.

3.1 Demographics

Time is assumed to be discrete, and the model period is equal to one year. The economy is populated by $J$ overlapping generations of ages $j = 20, 21, ..., J$, with $J$ being the maximum possible age an agent can live until. The size of each new cohort entering the economy grows at a constant rate $n$. Lifetime length is uncertain with mortality risk varying over the lifetime. Conditional on being alive at age $j$, $\Psi_j$ is the probability of an agent living to age $j + 1$. Since agents are not certain how long they will live they may die while still holding assets. If an agent dies with assets, the assets are confiscated by the government and distributed equally to all the living agents as transfers ($Tr_j$). All agents are required to retire at an exogenously set age $j_r$.

3.2 Individual

An individual is endowed with one unit of productive time per period that he divides between labor and leisure. An agent earns $w_\omega_i h_i j$ for their labor where $\omega_{i,j}$ is the idiosyncratic productivity of agent $i$ at age $j$. Agents split their income between saving with a one period risk free asset ($a_{i,j}$) and consumption. Agents choose labor, savings, and consumption in order to maximize his lifetime utility

$$u(c_{i,j}, h_{i,j}) + \sum_{s=1}^{J-j-1} \beta^s \prod_{q=1}^s (\Psi_q) u(c_{i,s+1}, h_{i,s+1}),$$

(35)
where \(c_{i,j}\) is consumption, and \(h_{i,j}\) is the hours spent providing labor services. Agents discount the next period’s utility by the product of \(\Psi_j\) and \(\beta\). \(\beta\) is the discount factor conditional on surviving, and the unconditional discount rate is \(\beta\Psi_j\).

The log of an agent’s idiosyncratic productivity \(\omega_{i,j}\) in the exogenous model can be split into four additively separable components,

\[
\log \omega_{i,j} = \varepsilon_j + \alpha_i + \nu_t + \theta_t. \tag{36}
\]

and in the LBD model,

\[
\log \omega_{i,j} = s_{i,j} + \alpha_i + \nu_t + \theta_t. \tag{37}
\]

In this specification, based on the estimates in Kaplan (2012) from the Panel Study of Income Dynamics (PSID), \(\varepsilon_j\) or \(s_{i,j}\) governs age-specific human capital. Moreover, \(\alpha \sim NID(0, \sigma^2_\alpha)\) is an individual-specific fixed effect (or ability) that is observed at birth and stays fixed for an agent over the life cycle, \(\theta_t \sim NID(0, \sigma^2_\theta)\) is a transitory shock to productivity received every period, and \(\nu_t\) is a persistent shock, which follows a first-order autoregressive process:

\[
\nu_t = \rho \nu_{t-1} + \psi_t \text{ with } \psi_t \sim NID(0, \sigma^2_\nu) \text{ and } \nu_1 = 0. \tag{38}
\]

In the exogenous model an agent’s age-specific human capital \(\varepsilon_j\) is exogenously determined. In the LBD model, an agent’s age-specific human capital, \(s_{i,j}\), is a function of a skill accumulation parameter, previous age-specific human capital, and time worked, denoted by \(s_{i,j} = S_{LBD}(\Omega_{j-1}, s_{i,j-1}, h_{i,j-1})\). \(\{\Omega_j\}_{j=1}^{h-1}\) is a sequence of calibration parameters that are set so that in the LBD model, under the baseline-fitted U.S. tax policy, the agents’ choices result in agents having the same age-specific human capital, on average, as in the exogenous model.

### 3.3 Market structure

The markets are incomplete and agents cannot fully insure against the idiosyncratic labor productivity and mortality risks by trading state-contingent assets. They can, however, partially self-insure against these risks by accumulating precautionary asset holdings, \(a\). The stock of assets earns a market return \(r_t\). I assume that households enter the economy with no assets and are not allowed to borrow against future income, so that \(a_{i,0} = 0\) and \(a_{i,j} \geq 0\) for all \(i\) and \(j\).
3.4 Firm

Firms are perfectly competitive with constant returns to scale production technology. Aggregate technology is represented by a Cobb-Douglas production function. The aggregate resource constraint is,

\begin{equation}
C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq K_t^\xi N_t^{1-\xi},
\end{equation}

where \(K_t\), \(C_t\), and \(N_t\) represent the aggregate capital stock, aggregate consumption, and aggregate labor (measured in efficiency units), respectively. Additionally, \(\xi\) is the capital share and \(\delta\) is the depreciation rate for physical capital. Unlike the analytically tractable model, I do not assume a linear production function in the computational model, so prices are determined endogenously and fluctuate with aggregate capital and labor.

3.5 Government Policy

The government has two fiscal instruments to finance its consumption, \(G_t\), which is done in an unproductive sector.\(^{20}\) First, the government taxes capital income, \(y_k \equiv r_t(a + Tr_t)\), according to a capital income tax schedule \(T^K[y_k]\). Second, the government taxes each individual’s taxable labor income. Part of the pre-tax labor income is accounted for by the employer’s contributions to social security, which is not taxable under current U.S. tax law. Therefore, the taxable labor income is \(y_l \equiv w_t s_j h_j (1 - .5\tau_{ss})\), which is taxed according to a labor income tax schedule \(T^l[y_l]\). I impose four restrictions on the labor and capital income tax policies. First, I assume human capital is unobservable and cannot be taxed directly. Second, I assume the tax rates cannot be age-dependent. Third, both of the taxes are solely functions of the individual’s relevant taxable income in the current period. Finally, I rule out the use of lump sum taxes.

In addition to raising resources for consumption in the unproductive sector, the government runs a pay-as-you-go (PAYGO) social security system. In this reduced-form social security program, the government pays \(SS_t\) to all individuals that are retired. Social security benefits are determined such that retired agents receive an exogenously set fraction, \(b_t\), of the average income of all working individuals. Social security is financed by taxing labor income at a flat rate, \(\tau_{ss,t}\). The payroll tax rate \(\tau_{ss,t}\) is set to assure that the social security system has a balanced budget each period. The social security system is not considered part of the tax policy that the government optimizes. I include this simplified social security program because excluding this program would cause an agent to overemphasize savings since all of their post-retirement

\(^{20}\text{Including } G_t \text{ such that it enters the agent’s utility function in an additively separable manner is an equivalent formulation.}\)
consumption would need to be financed with private savings.\footnote{Peterman (2013) demonstrates that excluding social security can have notable effects on the optimal tax policy.}

### 3.6 Definition of Stationary Competitive Equilibrium

In this section I define the stationary competitive equilibrium for the exogenous model. See appendix B for the definition of the competitive equilibriums in the LBD model. An agent’s state variables, \( x \), are assets \( a \), age \( j \), ability \( \alpha \), persistent shock \( \nu \), and idiosyncratic shock \( \theta \). For a given set of exogenous demographic parameters \( \{ n, \Psi_j \} \), a sequence of exogenous age-specific human capital \( \{ \varepsilon_j \}_{j=1}^{j_r-1} \), a government labor tax function \( T^l : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \), a government capital tax function \( T^k : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \), a social security tax rate \( \tau_{ss} \), social security benefits \( SS \), a production plan for the firm \( (N, K) \), and a utility function \( U : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \), a stationary competitive equilibrium consists of agents’ decision rules \( \{ c, h \} \) for each state \( x \), factor prices \( \{ w, r \} \), transfers \( Tr \), and the distribution of individuals \( \{ \mu(x) \} \) such that the following holds:

1. Given prices, policies, transfers, benefits, and that \( \omega \) follows equation 36, the agent maximizes equation 35 subject to

\[
\begin{align*}
    c + a' &= w\omega h - \tau_{ss}w\omega h + (1 + r)(a + Tr) - T^l[w\omega h(1 - .5\tau_{ss})] - T^k[r(a + Tr)], \\
    c + a' &= SS + (1 + r)(a + Tr) - T^k[r(a + Tr)],
\end{align*}
\]

for \( j < j_r \), and

\[
\begin{align*}
    c + a' &= SS + (1 + r)(a + Tr) - T^k[r(a + Tr)],
\end{align*}
\]

for \( j \geq j_r \).

Additionally,

\[
\begin{align*}
    c &\geq 0, 0 \leq h \leq 1, a \geq 0, a_1 = 0.
\end{align*}
\]

2. Prices \( w \) and \( r \) satisfy

\[
\begin{align*}
    r &= \zeta \left( \frac{N}{K} \right)^{1-\zeta} - \delta \\
    w &= (1 - \zeta) \left( \frac{K \setminus \zeta}{N} \right).
\end{align*}
\]

3. The social security policies satisfy

\[
SS = b\frac{wN}{\sum_{j<j_r} \mu(x)}
\]
4. Transfers are given by

\[ Tr = \sum (1 - \Psi) a' \mu(x). \]  

(47)

5. Government balances its budget

\[ G = \sum T_k [r(a + Tr)] \mu(x) + \sum_{j<j_r} T_l [w \omega h (1 - .5 \tau_{ss})] \mu(x). \]  

(48)

6. The market clears

\[ K = \sum a \mu(x), \]  

(49)

\[ N = \sum h \omega \mu(x), \]  

(50)

and

\[ \sum c \mu(x) + \sum a' \mu(x) + G = K^\zeta N^{1-\zeta} + (1 - \zeta)K. \]  

(51)

7. The distribution of \( \mu(x) \) is stationary, that is, the law of motion for the distribution of individuals over the state space satisfies \( \mu(x) = Q_\mu \mu(x) \), where \( Q_\mu \) is a one-period recursive operator on the distribution.

4 Calibration and Functional Forms

Prior to solving the models, it is necessary to choose functional forms and calibrate the models’ parameters. Calibrating the models involves a two-step process. The first step is choosing parameter values for which there are direct estimates in the data. These parameter values are in Table 1. Second, to calibrate the remaining parameters, values are chosen so that under the baseline-fitted U.S. tax policy certain targets in the model match the values observed in the U.S. economy (Simulated Method of Moments).\(^\text{22}\) These values are in Table 2.

\(^{22}\)Since these are general equilibrium models, changing one parameter will alter all the values in the model that are used as targets. However, I present targets with the parameter that they most directly correspond to.
Table 1: Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retire Age: $j_r$</td>
<td>66</td>
<td>By Assumption</td>
</tr>
<tr>
<td>Max Age: $J$</td>
<td>100</td>
<td>By Assumption</td>
</tr>
<tr>
<td>Surv. Prob: $\Psi_j$</td>
<td>Bell and Miller (2002)</td>
<td>Data</td>
</tr>
<tr>
<td>Pop. Growth: $n$</td>
<td>1.1%</td>
<td>Data</td>
</tr>
<tr>
<td><strong>Firm Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>.36</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta$</td>
<td>8.33%</td>
<td>$\frac{I}{P} = 25.5%$</td>
</tr>
<tr>
<td>$A$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td><strong>Productivity Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence Shock: $\sigma^2_v$</td>
<td>0.017</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Persistence: $\rho$</td>
<td>0.958</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Permanent Shock: $\sigma^2_{\alpha}$</td>
<td>0.065</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Transitory Shock: $\sigma^2_{\varepsilon}$</td>
<td>0.081</td>
<td>Kaplan (2012)</td>
</tr>
</tbody>
</table>

Adding endogenous human capital accumulation to the model fundamentally changes the model. Accordingly, if the calibration parameters are the same, then the value of the targets will be different in the LBD and exogenous models. To assure that both the models match the targets under the baseline-fitted U.S. tax policy, I calibrate the set of parameters based on targets separately in the two models. This calibration implies that these parameters are different in the exogenous and LBD models.

### 4.1 Demographics

In the model, agents are born at a real world age of 20 that corresponds to a model age of 1. Agents are exogenously forced to retire at a real world age of 66. If an individual survives until the age of 100, he dies the next period. I set the conditional survival probabilities in accordance with the estimates in Bell and Miller (2002). I adjust the size of each cohort’s share of the population to account for a population growth rate of 1.1 percent.

### 4.2 Preferences

Agents have time-separable preferences over consumption and labor services. Conditional on survival, agents discount their future utility by $\beta$. I use, $e^{(1-\sigma_1)\frac{h}{1-\sigma_1}} - \frac{(h)^{1+\sigma_2}}{1+\sigma_2}$, as the utility function for both models. This utility function is separable and homothetic in consumption and labor which implies that the Frisch elasticity
Table 2: Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Exog.</th>
<th>LBD</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional Discount: $\beta$</td>
<td>0.996</td>
<td>0.993</td>
<td>$K/Y = 2.7$</td>
</tr>
<tr>
<td>Risk aversion: $\sigma_1$</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Frisch Elasticity: $\sigma_2$</td>
<td>0.5</td>
<td>0.74</td>
<td>Frisch = $\frac{1}{2}$</td>
</tr>
<tr>
<td>Disutility of Labor: $\chi$</td>
<td>53.7</td>
<td>41.0</td>
<td>Avg. $h_j + n_j = \frac{1}{3}$</td>
</tr>
<tr>
<td>Government Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Upsilon_0$</td>
<td>.258</td>
<td>.258</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>$\Upsilon_1$</td>
<td>.768</td>
<td>.768</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>$G$</td>
<td>0.155</td>
<td>0.152</td>
<td>17% of $Y$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.5</td>
<td>0.5</td>
<td>CKK</td>
</tr>
</tbody>
</table>

in the exogenous model will be constant. Using a utility function that implies a constant Frisch elasticity in the exogenous model is convenient since it allows me to isolate the effects of the elasticity channel.

I determine $\beta$ such that the capital-to-output ratio is 2.7, in accordance with U.S. data.\textsuperscript{23} I determine $\chi$ such that under the baseline-fitted U.S. tax policy, agents spend on average one third of their time endowment in non-leisure activities.\textsuperscript{24} Following CKK, I set $\sigma_1 = 2$, which controls the relative risk aversion.\textsuperscript{25} Past micro-econometric studies (such as Altonji (1986), MaCurdy (1981), and Domeij and Flodén (2006)) estimate the Frisch elasticity to be between 0 and 0.5. However, more recent research has shown that these estimates may be biased downward. Reasons for this bias include: utilizing weak instruments; not accounting for borrowing constraints; disregarding the life cycle effect of endogenous-age specific human capital; omitting correlated variables such as wage uncertainty; and not accounting for labor market frictions.\textsuperscript{26} Therefore, I set $\sigma_2$ such that the Frisch elasticity is at the upper bound of the range (0.5).

4.3 Idiosyncratic Productivity

Following Huggett and Parra (2010), the process for the idiosyncratic labor productivity shocks are calibrated based on the estimates from the PSID data in Kaplan (2012).\textsuperscript{27} The permanent, persistent, and transitory idiosyncratic shocks to individual’s productivity are distributed normal with a mean of zero. The remaining parameters are also set in accordance with the estimates in Kaplan (2012): $\rho = 0.958$, $\sigma_{\alpha}^2 = 0.065$.

\textsuperscript{23}This is the ratio of fixed assets and consumer durable goods, less government fixed assets to GDP (CKK).

\textsuperscript{24}Using a target of one-third is standard in quantitative exercises. For examples, see CKK, Nakajima (2010), and Garriga (2001).

\textsuperscript{25}Even though CKK use a different utility specification, their specification has a parameter that corresponds to $\sigma_1$.

\textsuperscript{26}Some of these studies include Imai and Keane (2004), Domeij and Flodén (2006), Pistaferri (2003), Chetty (2009), and Contreras and Sinclair (2008).

\textsuperscript{27}For details on estimation of this process, see Appendix E in Kaplan (2012).
\(\sigma_\nu^2 = 0.017\) and \(\sigma_\epsilon^2 = 0.081\). I discretize all three of the shocks in order to solve the model, using two states to represent the transitory and permanent shocks and five states for the persistent shock. For expositional convenience, I refer to the two different states of the permanent shock as high and low ability.

### 4.4 Age-Specific Human Capital

The age-specific human capital parameters that require calibration are different in the exogenous and LBD models. In the exogenous model, I set \(\{\epsilon_j\}_{j=0}^{b-1}\) to be consistent with the values estimated in Kaplan (2012) which are based off of hourly earnings in the Panel Survey of Income Dynamics.\(^{28}\)

In the LBD model I use the same functional form for human capital accumulation as in Hansen and İmrohoroğlu (2009),

\[
s_{j+1} = \Omega_j \Phi_1 s_{j}^j \Phi_2, \tag{52}
\]

where \(s_j\) is the age-specific human capital for an agent at age \(j\), \(\Omega_j\) is an age-specific calibration parameter, \(\Phi_1\) controls the importance of an agent’s current human capital on LBD, and \(\Phi_2\) controls the importance of time worked on LBD. I do not set \(\{s_i, j\}_{j=0}^{b-1}\) directly, rather I calibrate the sequence \(\{\Omega_j\}_{j=1}^{b-1}\) such that the agents’ equilibrium labor choices lead the average \(\{s_i, j\}_{j=0}^{b-1}\) under the baseline-fitted U.S. tax code to match the age-specific human capital calibrated in the exogenous model (\(\{\epsilon_j\}_{j=0}^{b-1}\)).\(^{29}\) To calibrate the rest of the LBD parameters, I rely on the estimates in Chang et al. (2002), setting \(\Phi_1 = 0.407\) and \(\Phi_2 = 0.326.\(^{30}\)

### 4.5 Firm

I assume the aggregate production function is Cobb–Douglas. The capital share parameter, \(\zeta\), is set at \(0.36.\) The depreciation rate is set to target the observed investment output ratio of 25.5 percent. These parameters are summarized in Table 1.

### 4.6 Government Policies and Tax Functions

While calibrating parameters such that targets in the models match the values in the data it is important to use a baseline tax function that mimics the U.S. tax code. I use the estimates of the U.S. tax code in Gouveia

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\(^{28}\)I make three adjustments to the process. The profile is smoothed by fitting a quadratic function in age, normalized such that the value equal one when an agent enters the economy, and extended to cover ages 20 through 66 . The estimates in Kaplan (2012) are available for ages 25-65.

\(^{29}\)I calibrate \(\{\Omega_j\}_{j=1}^{b-1}\) such that the sequence is smooth over the life cycle.

\(^{30}\)This functional form implies full depreciation of skills if individuals choose not to work in the LBD model. However, full depreciation will never be binding because agents choose to work large quantities in all periods in the exogenous model which does not include the additional human capital incentive for working.
and Strauss (1994) for this tax policy, which I refer to as the baseline-fitted U.S. tax policy. The authors match the U.S. tax code to the data using a three parameter functional form,

\[ T(y; \gamma_0, \gamma_1, \gamma_2) = \gamma_0 (y - (y^{\gamma_1} + \gamma_2)^{\frac{1}{\gamma_1}}), \]  

where \( y \) represents the sum of labor and capital income. The average tax rate is principally controlled by \( \gamma_0 \), and \( \gamma_1 \) governs the progressivity of the tax policy. To ensure that taxes satisfy the budget constraint, \( \gamma_2 \) is left free. Gouveia and Strauss (1994) estimate that \( \gamma_0 = .258 \) and \( \gamma_1 = .768 \) when fitting the data. The authors do not fit separate tax functions for labor and capital income. Accordingly, I use a uniform tax system on both sources of income for the baseline-fitted U.S. tax policy.

I calibrate government consumption, \( G \), so that it equals 17 percent of output under the baseline-fitted U.S. tax policy, as observed in the U.S. data.\(^{31}\) This approach implies that \( \gamma_2 \) is determined as the value that equates government spending to 17 percent of GDP. When searching for the optimal tax policy, I restrict my attention to revenue neutral changes that imply that government consumption is equal under the baseline-fitted U.S. tax policy and the optimal tax policy. In addition when searching for the optimal tax policy I allow the tax policy on capital and labor to be different.

In addition to government consumption, the government also runs a balanced-budget social security program. Social security benefits are set so that the replacement rate, \( b \), is 50 percent.\(^{32}\) The payroll tax, \( \tau_{ss} \), is determined so that the social security system is balanced each period.

### 5 Computational Experiment

The computational experiment is designed to determine the tax policy that maximizes a given social welfare function. I choose a social welfare function (SWF) that corresponds to a Rawlsian veil of ignorance (Rawls (1971)). The social welfare is equivalent to maximizing the ex-ante expected lifetime utility of a newborn. When searching for the optimal tax policy I restrict my attention to labor income taxes of the three parameter functional form from Gouveia and Strauss (1994) and flat capital taxes.\(^{33}\) Therefore the computational

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\(^{31}\)To determine the appropriate value for calibration, I focus on government expenditures less defense consumption.

\(^{32}\)The replacement rate matches the rate in CKK and Conesa and Krueger (2006). The Social Security Administration estimates that the replacement ratio for the median individual is 40 percent (see Table VI.F10 in the 2006 Social Security Trustees Report; available at www.ssa.gov/OACT/TR/TR06/). This estimate is lower than the replacement rate I use; however, if one also includes the benefits paid by Medicare, then the observed replacement ratio would be higher.

\(^{33}\)I tested the effect of the restriction that the capital tax must be flat in a model without within cohort heterogeneity and found the restriction had no effect on the optimal tax policy in either the exogenous or LBD models.
experiment is maximizing the expected utility for a newborn,
\[
SWF(\tau_{h0}, \tau_{h1}, \tau_{h2}, \tau_k) = \mathbb{E} \left[ u(c_1, h_1) + \sum_{s=1}^{J-j-1} \prod_{q=1}^{\beta_s} (\Psi_q) u(c_{s+1}, h_{s+1}) \right],
\]
(54)
where \(\tau_{h0}, \tau_{h1}, \) and \(\tau_{h2}\) are the labor income tax parameters, and \(\tau_k\) is the flat tax rate on capital income. To determine the effects of endogenous human capital accumulation, I compare the tax policies that maximize the SWF in the two models.

6 Results

In this section I quantitatively assess the effects on the optimal tax policy of including LBD in a calibrated life cycle model. In order to assess LBD’s effect, I determine the optimal tax policies in the exogenous and LBD models and also highlight the channels that cause the differences. To fully understand the effects of endogenous human capital accumulation, I analyze the aggregate economic variables and life cycle profiles in both models under the baseline-fitted U.S. tax policy as well as the changes induced by implementing the optimal tax policies.

6.1 Optimal Tax Policies in Exogenous and LBD Models

Table 3 describes the optimal tax policy parameters and Figure 3 plots the average and marginal labor tax rates by income in both models. Starting with the exogenous model, the optimal tax policy is a 25.2 percent tax on capital and approximately a 33.5 percent tax on labor income with a deduction of $7,500.\textsuperscript{34,35} Unlike the analytically tractable model, the optimal tax on capital in the computational exogenous model is positive. The additional aspects in the computational exogenous model that motivate a positive optimal capital tax include: the inability of the government to borrow, agents being liquidity constrained, the government not being able to tax transfers at a separate rate from ordinary capital income, and exogenous retirement coupled with social security (see Peterman (2013) for a thorough discussion).\textsuperscript{36} Moreover, unlike the analytically tractable model, the optimal tax on capital in the computational model is positive. The additional aspects in the computational exogenous model that motivate a positive optimal capital tax include: the inability of the government to borrow, agents being liquidity constrained, the government not being able to tax transfers at a separate rate from ordinary capital income, and exogenous retirement coupled with social security (see Peterman (2013) for a thorough discussion).\textsuperscript{36}

\textsuperscript{34}When solving for the optimal labor income tax policies I restrict my attention to the three parameter functional form in Gouveia and Strauss (1994). I find that, similar to other studies (see CKK), the optimal policies can be closely approximated by a flat tax with a fixed deduction. Therefore, for expositional convenience I describe the optimal labor tax policies in this manner.

\textsuperscript{35}Comparing the optimal tax policy in the exogenous model and the optimal tax policy in CKK, the labor tax deduction is similar but the optimal tax rate on capital (labor) is smaller (larger) in the exogenous model. The difference in the relative capital and labor tax rates are because the exogenous model uses a utility function that is homothetic and separable in both consumption and labor. This type of utility function tends to decrease the optimal capital tax (see Peterman (2013) for further discussion).

\textsuperscript{36}I choose to include some of these features so that incentives in the model correspond to the incentives in the U.S. economy. For example, the reduced form social security program is necessary so that the level of individual savings are realistic. Aligning the savings incentives are important since the capital to output ratio is a target used to calibrate the discount rate. Additionally, liquidity constraints are included in the model to capture that a sizeable portion of the population face borrowing constraints (see Jappelli
tractable model, the computational model includes within cohort heterogeneity. Therefore, the social planner not only considers efficiency, but determines the optimal amount of progressivity by weighing the trade off between equity (progressive tax) and efficiency (flat tax). I find that in the exogenous model the social planner is willing to distort agents’ decisions with a progressive labor income tax in order to redistribute and provide social insurance.\(^{37}\) Instead of focusing on the motives for these differences between the optimal tax policies in the analytically tractable exogenous model and computational exogenous model, this paper treats the optimal tax policy in the computational exogenous model as the benchmark for comparison with the optimal tax policies in the LBD model.

Table 3: Optimal Tax Policies in Benchmark Models

<table>
<thead>
<tr>
<th>Tax Rate</th>
<th>Exog</th>
<th>LBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_{h0})</td>
<td>0.335</td>
<td>0.209</td>
</tr>
<tr>
<td>(\tau_{h1})</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>(\tau_{h2})</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>(\tau_{k})</td>
<td>0.252</td>
<td>0.299</td>
</tr>
</tbody>
</table>

Note: \(\tau_{h2}\) is in trillions. The large value of \(\tau_{h2}\) implies that the labor tax function mimics a flat tax with a fixed deduction.

Focusing on the effect of adding LBD in the computational model, the optimal tax policy in the LBD model is a 29.8 percent flat tax on capital and a 20.9 percent flat tax on labor income. There are two main changes to the optimal tax policy caused by adding LBD. First, a progressive labor income tax is no longer optimal in the LBD model. Second, the optimal tax on capital increases by 4.7 percentage points (19 percent) with LBD.

Adding LBD alters the optimal tax policy through two channels. First, as Section 2.2.2 demonstrates, adding LBD creates an additional intertemporal link. In particular, decisions with regards to current labor affect the level of future human capital. The distortions from a progressive labor income tax are enhanced in the LBD model because they are transmitted through this additional intertemporal link. For example, in the LBD model, a progressive tax implies that working more today will increase human capital leading to a higher future marginal labor income tax rate. Therefore, adding LBD will tend to lead the social planner to be more reluctant to use a non-flat labor income tax policy.

\(^{37}\)CKK demonstrate that the progressive tax is optimal due to the heterogeneity. Moreover, I confirm this results by solving for the optimal tax policy in my exogenous model when I eliminate any within cohort heterogeneity and find that in this version of the model the optimal labor income tax is no longer progressive.

(1990)). Moreover, other of these features, such as accidental bequests, are included to close the model in a tractable manner.
Second, as Section 2.2.1 demonstrates, adding LBD causes agents to supply labor relatively more elastically as they age because the human capital benefit decreases. This change in the Frisch labor supply elasticity is apparent in the average Frisch labor supply elasticity profile from the models (see Figure 2). Because the Frisch elasticity tends to increase as an agent ages in the LBD model the social planner would like to tax labor income earned when an agent is young at a relatively higher rate than income earned when an agent is old. The social planner can mimic this type of age-dependent tax with either a tax on capital or a progressive/regressive tax on labor income. A positive tax on capital implies that the tax on labor income is monotonically decreasing as an agent ages. In contrast, the effectiveness of mimicking this type of age-dependent tax policy with a progressive/regressive labor income tax depends on the shape of the average labor earnings profile. Figure 3 plots the average labor earnings by age in the LBD model. Although average labor income is increasing over a slight majority of the working lifetime, the income profile is humped shaped. The non-monotonicity of the labor income profile implies that although a more regressive labor income tax will tend to tax labor income earned when an agent is young at a higher rate, it will also tax labor income earned at the end of the working lifetime at a relatively higher rate.

In order to decompose how much of the two changes in the optimal tax policies are from the elasticity

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38The Frisch elasticity profile in the LBD model rises rapidly in the last period before retirement due to the function form of the human capital accumulation equation. When testing the effect of the elasticity channel I generally find that excluding the increase in the period before retirement has no impact.
Figure 2: Frisch Elasticity

Figure 3: Labor Earnings in LBD Model
channel versus the inclusion of an additional intertemporal link I determine the optimal tax policy in an alternative exogenous model. In the altered exogenous model I vary $\sigma_2$ by age such that the shape of the average lifetime Frisch labor supply elasticity profile is the same as the shape in the LBD model. Solving for the optimal tax policy in this altered exogenous model isolates the effect of the elasticity channel because the exogenously varying Frisch elasticity incorporates this channel but without incorporating the additional intertemporal link.

I find that the optimal tax on capital in this altered exogenous model is 30.8 percent and the optimal labor tax is approximated by a flat 31.6 percent tax on labor with a deduction of $7,750. The similar optimal tax on capital in both the altered exogenous model and the LBD model indicates that the elasticity channel is primarily responsible for the increase in the optimal capital tax when LBD is included.

Given that the average labor income profile is humped shaped, it is not surprising that the social planner finds a tax on capital, as opposed to a regressive labor income tax, as the more effective way to mimic age-dependent taxes. Moreover, the similar progressivity of the labor income tax in the benchmark exogenous model and the altered exogenous model demonstrates that the elasticity channel is not responsible for the change in the optimal progressivity of the labor income tax when LBD is included. Instead, the additional intertemporal link in the LBD model is responsible for altering the optimal progressivity of the labor income tax. In particular, LBD enhances the distortion from a progressive labor tax which causes it to no longer be optimal for the social planner to use a progressive tax to redistribute in the LBD model.

Next, I turn to the welfare effects of not accounting for LBD when solving for the optimal tax policy. I measure welfare in consumption equivalent variation (CEV) which is defined as the uniform increase in expected consumption an agent would need in expectation at each age in order to be indifferent to being born into an economy with a less optimal tax policy. In particular, I am interested in the welfare gains in the LBD model associated with adopting the true optimal tax policy (which includes a flat tax on labor income and a larger tax on capital) versus implementing the optimal tax policy solved for in the exogenous model (which includes a progressive labor tax and a lower tax on capital). The first row of Table 4 describes the average welfare effects of this change for all agents, low productivity agents, and high productivity agents, respectively. I find that shifting from the sub-optimal tax policy solved for when not accounting for LBD to the true optimal causes an average increase in welfare that is equivalent to 4.1 percent of expected lifetime

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39. In order to keep the altered exogenous model consistent I use the same calibration parameters as in the benchmark exogenous model and also hold transfers and social security benefits constant between the two models. I choose to keep these constant because Peterman (2013) demonstrates that changes in both can have large impacts on the optimal tax policy.

40. The Frisch elasticity profile in the LBD model rises rapidly in the last few period before retirement due to the function form of the human capital accumulation equation. I find that the effect of the elasticity channel is similar in an altered exogenous model without the increase in the Frisch elasticity over the last few periods. Therefore, it is mainly the upward slope of the profile over the whole life and not just the last few periods that is responsible for affecting the optimal tax policy.
consumption. This increase tends to be larger for high ability agents than for low ability agents because the high ability agents particularly benefit from the elimination of the progressivity of the labor income tax. The second row and third row of Table 4 decompose how much of these welfare changes are from the change in the progressivity of the labor tax and how much is from the change in the relative ratio of capital to labor taxes. I find that almost all of these welfare gains come from eliminating the progressive labor income tax as opposed to adjusting the relative levels of the capital and labor income tax. These results demonstrate that in the LBD model there are particularly large welfare consequences to distorting decisions with a sub-optimal level of progressivity in the labor income tax.

Table 4: Welfare Gains in LBD Model Compared to Misspecified Optimal Tax Policy

<table>
<thead>
<tr>
<th>Change</th>
<th>Average</th>
<th>Low Types</th>
<th>High Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>4.1%</td>
<td>2.7%</td>
<td>6.0%</td>
</tr>
<tr>
<td>Removing Progressivity</td>
<td>4.0%</td>
<td>2.7%</td>
<td>5.9%</td>
</tr>
<tr>
<td>Remainder</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Note: Welfare gains are solved for in the LBD model and are relative to welfare in the LBD when the tax policy is the optimal tax policy solved for in the exogenous model. The first row determines the welfare gained shifting to the true optimal tax policy. The second row determines the welfare gained from adopting a flat tax policy but holding the relative revenue gained from the capital and labor tax the same as in the optimal tax policy solved for in the exogenous model. The third row is the difference between the first two rows.

6.2 The Effects of Adding Endogenous Age-Specific Human Capital

This section analyzes the effect on the aggregate economic variables and life cycle profiles from adding LBD to the exogenous model under the baseline-fitted U.S. tax policy. Figure 4 plots the life cycle profiles of hours, consumption, assets, and age-specific human capital in both models. Table 5 summarizes the aggregate economic variables under both the baseline-fitted U.S. tax policy and optimal tax policies.

Comparing the exogenous and LBD models under the baseline fitted U.S. tax policy, the first and fourth columns of Table 5 demonstrate that the levels of aggregate hours, labor supply, and aggregate capital are similar in the two models. The calibrated parameters are determined so that under the baseline-fitted U.S. tax policy the models match certain targets from the data. Since many of the aggregate economic variables

41The second row isolates the effect of the progressive tax by determining how much welfare is gained by removing the progressive labor income tax in the misspecified tax policy but setting the flat capital and labor tax rates such that the relative amount of government revenue raised from each source does not change. The third row is the difference between the first two rows, or the remainder of the welfare gains which are attributed to setting the optimal relative capital to labor tax rates.
Table 5: Aggregate Economic Variables

<table>
<thead>
<tr>
<th>Aggregate</th>
<th>Exogenous</th>
<th>LBD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Optimal</td>
</tr>
<tr>
<td>Y</td>
<td>0.91</td>
<td>0.88</td>
</tr>
<tr>
<td>K</td>
<td>2.46</td>
<td>2.35</td>
</tr>
<tr>
<td>N</td>
<td>0.52</td>
<td>0.51</td>
</tr>
<tr>
<td>Avg Hours</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>w</td>
<td>1.12</td>
<td>1.11</td>
</tr>
<tr>
<td>r</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Tr</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Tax Rate</th>
<th>Baseline</th>
<th>Optimal</th>
<th>Baseline</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>22.2%</td>
<td>18.6%</td>
<td>22.3%</td>
<td>20.9%</td>
</tr>
<tr>
<td>Capital</td>
<td>20.7%</td>
<td>25.2%</td>
<td>20.8%</td>
<td>29.9%</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.07</td>
<td>0.74</td>
<td>1.07</td>
<td>0.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marginal Tax Rate</th>
<th>Baseline</th>
<th>Optimal</th>
<th>Baseline</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>21.5%</td>
<td>31.9%</td>
<td>21.6%</td>
<td>20.9%</td>
</tr>
<tr>
<td>Capital</td>
<td>20.1%</td>
<td>25.2%</td>
<td>20.1%</td>
<td>29.9%</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.07</td>
<td>1.27</td>
<td>1.07</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Note: The average hours refers to the average percent of time endowment worked in the productive labor sector. Since the marginal tax rates vary with income, the reported marginal tax rates are the population weighted average marginal tax rates for all agent.
are targets and these calibration parameters are determined separately in the exogenous and LBD models, the aggregates are similar in the two models.

Figure 4: Life Cycle Profiles under Baseline-Fitted U.S. Tax Policy

Labor Supply

Consumption

Savings

Age-specific Human Capital

Note: These plots are life cycle profiles of the three calibrated models under the baseline-fitted U.S. tax policy.

Although adding LBD does not have a large effect on the aggregate economic variables, it does cause changes in the life cycle profiles. Adding LBD causes agents to work relatively more at the beginning of their working life when the human capital benefit is larger, and less later when the benefit is smaller (see the solid black and solid red lines in the upper-left panel of Figure 4). The upper-right panel shows that the lifetime consumption profile is flatter in the LBD model compared to the exogenous model. The intertemporal Euler equation controls the slope of consumption profile over an agent’s lifetime. The relationship is

$$\left( \frac{c_{j+1}}{c_j} \right)^{\sigma_i} = \psi_j \beta \bar{r}_t,$$

where $\bar{r}_t$ is the marginal after-tax return on capital. In order to induce the same capital to output ratio in the LBD model, $\beta$ is lower which leads to the flatter consumption profile. Moreover, the lower value of $\beta$ in the LBD model decreases the value an agent places on their consumption in future periods so agents’
savings are also relatively smaller for the second half of their lifetime (see the lower-left panel). The lifetime age-specific human capital profiles are similar in the two models since the sequence of parameters \( \{\Omega_j\}_{j=1}^{8} \) is calibrated so that age-specific human capital matches (see the lower-right panel of Figure 4).

### 6.3 The Effects of the Optimal Tax Policy in the Exogenous Model

This section examines the effects on the economy of adopting the optimal tax policy in the exogenous model. In the exogenous model, the optimal capital tax is larger than the average marginal tax under the baseline-fitted U.S. tax policy so adopting the optimal tax policy causes a decrease in aggregate capital (see columns one and two of Table 5). The average marginal labor tax is also larger under the optimal tax policy so adopting the optimal tax policy causes the labor supply to decrease. The decrease in labor supply is relatively smaller than the decrease in capital so the rental rate on capital increases and the wage rate decreases.

Figure 5: *Life Cycle Profiles in the Exogenous Model*

Figure 5 plots the life cycle profiles for time worked, consumption, and assets in the exogenous model under the baseline-fitted U.S. tax policies and the optimal tax policies. The solid lines are the profiles under

Note: Since the skills are the same in the exogenous models under the baseline-fitted U.S. tax policy and optimal tax policy, they are not plotted.
the baseline-fitted U.S. tax policies, and the dashed lines are the profiles under the optimal tax policies. Adopting the optimal tax policy in the exogenous model causes changes in all three life cycle profiles: (i) agents work less, especially early in their life, (ii) agents save less, and (iii) the lifetime consumption profile is flatter. The first change, agents working less early in their life, is a result of the higher implicit tax on young labor income due to the increase in the tax rate on capital income.

Implementing the optimal tax policy causes an increase in both the capital tax and the rental rate on capital leading to shifts in both the consumption and savings profiles. Overall, the increase in the capital tax dominates so the marginal after-tax return on capital falls under the optimal tax policy. Since the after-tax return controls the slope of the consumption profile, the profile is flatter under the optimal tax policy (Figure 5, upper-right panel). Moreover, because of the lower returns agents hold less savings (see the lower left panel of Figure 5).

6.4 The Effects of Optimal Tax Policy in the LBD Model

Adopting the optimal tax policy in the LBD model causes an increase in the capital tax, a decrease in the labor tax, and the removal of the progressive labor tax (see column four, five, and six of Table 5). Since adopting the optimal tax policies removes the inefficient progressive labor tax, the economy increases
without these distortions. Thus, both aggregate capital and labor increase. However because the marginal labor tax rate falls, while the marginal capital tax increases, aggregate labor increases more than aggregate capital. The relatively larger rise in labor translates into an decrease in the wage rate and a increase in the rental rate on capital.

Implementing the optimal tax policies in the LBD model causes agents to shift time worked from earlier to later years in response to the larger capital tax, which implicitly taxes labor income from early years at a higher rate. However, agents also increase their labor supply because the the distortions from the progressive income tax are eliminated. Taken as a whole, these two changes partially offset and the overall changes in labor supply tend to be smaller in the LBD model (upper-left panel of Figure 6) compared to the exogenous model. Because agents work more in their middle years, age-specific human capital is also higher for middle aged agents (see the lower-right panel). Applying the optimal tax policy introduces two opposing effects on the agent’s lifetime asset profile. First, agents increase their savings under the optimal tax policy because the economy is larger and the return to capital increases. Second, the larger capital tax under the optimal tax policy decreases the average marginal after-tax return on capital, causing agents to hold fewer assets. The first effect is constant for all agents. The second effect is not constant for all agents, but it is negatively proportional to an agent’s capital income because the baseline-fitted U.S. tax policy is progressive and the optimal tax policy is flat. This second effect dominates for younger and older agents and they tend to save less under the optimal tax policy. Conversely, the first effect dominates for middle-aged agents and they tend to save more. With regards to the consumption profile, these changes to the after-tax return to capital causes the consumption profile to steepen for middle-aged agents and flatten for younger and older agents (see the upper-right panel).  

7 Sensitivity of Optimal Tax Policy to Shape of Labor Supply Profile

In this section, I compare the exogenous model’s predictions for the life cycle profiles to the data and test the relationship between the shape of the labor supply profile and the optimal tax. I examine this relationship because the predicted labor supply profile in the exogenous model does not match the observed labor supply profile in the data. Moreover, the predicted labor supply profiles in the exogenous and LBD model are different, especially for older agents.

42To be precise, the consumption profile slopes downwards at a faster rate for older agents.
7.1 Comparison of Model to Data

Figure 7 plots the average life cycle profiles from the exogenous model under the baseline-fitted U.S. tax policy and in the data.\textsuperscript{43} The upper left panel compares the average percent of the time endowment that is spent working over the lifetime and the upper right compares the labor income. The actual labor supply and labor earnings profiles are constructed from the 1967 - 1999 waves of the Panel Survey of Income Dynamics (PSID). In the data I focus my attention on the labor supply and labor earnings for the head of the household between ages 20 and 80. The lower left panel compares the consumption profile in the model to the per-capita expenditures on food in the PSID. The lower right panel examines savings in the model and median total wealth in the 2007 Survey of Consumer Finances (SCF) for individuals between the ages of 20 and 80.\textsuperscript{44} I smooth through some of the volatility in the wealth data by using five year age bins.\textsuperscript{45}

Focusing on the labor supply profiles, the model’s predicted profile has a different general pattern than the data. In the data, the labor supply profile is humped shaped. In contrast, the model tends to predict the labor supply will decrease throughout the working lifetime. The model severely over predicts the amount of time young agents spend working because in the model agents cannot borrow against future earnings. Therefore, in order to ensure they can smooth consumption over their lifetime in case of a low idiosyncratic productivity shock, agents work more early in their life in order to accumulate precautionary savings.\textsuperscript{46} In contrast, in the data, some young households may have a means to borrow, minimizing the severity of this effect. As agents age and these constraints loosen, the labor supply profiles still do not match as the profile for prime-aged individuals is much flatter in the data. Because the shape of the predicted labor supply does not match the data, I check whether the shape of the labor supply profile affects the optimal tax policy in Section 7. I find that overall there is minimal connection between the shape of the labor supply profile and the optimal tax policy.\textsuperscript{47}

Focusing on the upper right panel, the earnings profile in the data is similar to the one generated by the model. Both profiles are humped shaped with a peak around forty years old. However, since agents are

\textsuperscript{43}Earnings, consumption, and savings from the model are converted to real 2012 dollars by equating the average earnings in the model and the data.

\textsuperscript{44}In order to prevent the upper tail of the wealth distribution from skewing the statistic for comparison, I choose to focus on the median level of wealth as opposed to the average.

\textsuperscript{45}The data for individuals after age 80 were not included because there were few observations in the sample leading the smoothed estimates to be extremely volatile.

\textsuperscript{46}For further discussion see Heathcote et al. (2010)

\textsuperscript{47}The lack of relationship between the labor supply profile and optimal tax policy is not surprising. Peterman (2013), Garriga (2001), Erosa and Gervais (2002) all demonstrate that when using a utility function that is homothetic and separable in labor and consumption, such as the one in the benchmark model, that regardless of the labor supply profile the government does not want to condition taxes on age. However, if the utility function is not homothetic and separable then the government wants to condition labor income taxes on age and in the absence of the ability to use age-dependent taxes a downward sloping labor supply will lead to a positive optimal tax on capital.
forced to retire at 65 in the model, but in the U.S. economy some head of households retire after the age of 65, the earnings profile for these older households are higher in the data.

When comparing the consumption profiles, I find that both profiles are hump-shaped. However, I find that consumption on food tends to peak earlier in the data than total consumption in the model. Additionally, comparing the growth in consumption from the age 20 to the peak, the model exhibits more growth in consumption over the lifetime. One possible reason for these differences is that the PSID data are limited to just expenditures on food while the model generated consumption represents all consumption.

Finally, I find that the savings profiles are similar in the model and the data. Both are hump-shaped, peaking around $300,000 at the age of 60. One discrepancy between the two profiles is that the model predicts that agents will deplete their savings more quickly than they do in the data. This discrepancy could arise because the model does not include any motive for individuals leaving a bequest for younger generations or holding savings in case of unexpected end of life expenditures such as medical expenses.

Figure 7: **Actual and Exogenous Life Cycle Profiles**

Note: These plots are life cycle profiles in the exogenous model under the baseline-fitted U.S. tax policy and the actual profiles in the data. The units of the consumption, earnings, and capital profiles are converted to real dollars by matching the average labor earnings in the model and in the data.
7.2 Sensitivity

In order to test whether the shape of the labor supply profile affects the optimal tax policy, I determine the optimal tax policy in an alternative exogenous model in which I vary the values of $\chi$ over the lifetime such that the average labor supply profile matches more closely the profile observed in the data. Figure 8 plots the labor supply profile generated in the exogenous benchmark model (solid black line) and the average hours worked in the data (solid blue line) under the baseline-fitted U.S. tax policy. Additionally, the dashed black line plots the labor supply generated in the alternative exogenous model which is calibrated to more closely match the actual labor supply profile. I find that the optimal tax policy in this alternative exogenous model ($\tau_{h0} = .33$, $\tau_{h1} = 20$, $\tau_{h2} = 100$, $\tau_k = .254$) is almost identical to the optimal tax policy in the benchmark exogenous model. These results indicate that the optimal tax policy is generally not related to the shape of the labor supply profile. These results are not surprising since one feature of the benchmark utility function is that it is homothetic and separable in labor and consumption. Therefore, labor supply is not related to the Frisch labor supply elasticity. This utility function eliminates the most active channel by which the labor supply profile affects the optimal tax policy.

Figure 8: Labor Supply Profiles
8 Conclusion

Two important questions for optimal taxation are at what rate should capital be taxed and should the income tax policy be progressive. In this paper I examine the effect of LBD on optimal taxation and find that it affects the answers to both questions. Analytically, I demonstrate that including endogenous human capital accumulation creates a motive for the government to condition labor income taxes on age and if disallowed either a non-zero capital tax or progressive/regressive labor income tax can be used to mimic these age-dependent taxes. Although a progressive/regressive tax can be used to mimic an age-dependent tax, I show that the distortions from this type of tax are enhanced in the LBD model, making it a less appealing option.

Quantitatively, I find that these channel cause the inclusion of LBD to change both the optimal mix of capital to labor taxes and also the optimal progressivity of the labor income tax. Specifically, I find that with exogenous age-specific human capital accumulation the optimal tax policy is a 25.2 percent tax on capital and 33.5 percent tax on labor with a deduction of $7,500. In contrast, I find that in the LBD model the optimal tax on capital increases to 29.9 percent. Moreover, the optimal labor income tax in the LBD model is a flat 20.9 percent tax.

LBD increases the motive for a capital tax since it alters the lifetime labor supply elasticity profile. Adding LBD to the model causes younger agents to supply labor relatively less elastically since the human capital benefit decreases over an agent’s lifetime. Moreover, because of the additional intertemporal link between current labor and future human capital, the social planner is no longer willing to use a progressive labor income tax in the the LBD model. Overall, I find that the welfare effects of not accounting for LBD when solving for the optimal tax policy are quite large (4.1 percent CEV). Therefore, including endogenous human capital accumulation has economically significant effects on the optimal tax policy. Given the large differences between the optimal tax policies in the two models and the large size of their welfare implications, these results indicate that when examining optimal taxation accurately incorporating the skills accumulation process is of first order importance.
A Analytical Derivations

A.1 Exogenous

The Lagrangian for this specification is

\[ \mathcal{L} = \frac{c_{1,t}}{1 - \sigma_1} - \frac{h_{1,t}}{1 + \frac{1}{\sigma_2}} + \beta \frac{c_{2,t+1}}{1 - \sigma_1} - \frac{h_{2,t+1}}{1 + \frac{1}{\sigma_2}} \]

\[ - \rho_t (c_{1,t} + c_{2,t} + K_{t+1} - K_t + G_t - rK_t - w(h_{1,t} + h_{2,t}w_t)) \]

\[ - \rho_{t+1} \theta (c_{1,t+1} + c_{2,t+1} + K_{t+1} - K_t + G_t + rK_t - w(h_{1,t+1} + h_{2,t+1}w_t)) \]

\[ + \lambda_t (c_{1,t} - \beta c_{2,t+1} - \chi h_{1,t} - \beta \chi h_{2,t+1}) \]

where \( \rho \) is the Lagrange multiplier on the resource constraint and \( \lambda \) is the Lagrange multiplier on the implementability constraint. The first order conditions with respect to labor, capital, and consumption are

\[ \rho_t = \chi h_{1,t}^{\frac{\sigma_t}{\sigma_2}} (1 + \lambda_t (1 + \frac{1}{\sigma_2})) \]  \( (57) \)

\[ \rho_{t+1} \theta w_t = \beta \chi h_{2,t+1}^{\frac{1}{\sigma_2}} (1 + \lambda_t (1 + \frac{1}{\sigma_2})) \]  \( (58) \)

\[ \rho_t = \theta (1 + r) \rho_{t+1} \]  \( (59) \)

\[ \rho_t = c_{1,t}^{\sigma_t} + \lambda_t (1 - \sigma_t) c_{1,t}^{-\sigma_t} \]  \( (60) \)

and

\[ \theta \rho_{t+1} = \beta c_{2,t+1}^{\sigma_t} + \beta \lambda_t (1 - \sigma_t) c_{2,t+1}^{-\sigma_t}. \]  \( (61) \)

Combining the first order equations for the governments problem with respect to capital and consumption yields

\[ \left( \frac{c_{2,t+1}}{c_{1,t}} \right)^{\sigma_t} = \frac{\beta \rho_t}{\rho_{t+1} \theta} \]  \( (62) \)

where \( \rho \) is the Lagrange multiplier on the resource constraint and \( \lambda \) is the Lagrange multiplier on the implementability constraint. Taking the ratio of the agent’s first order conditions, equations 5 and 6 under the benchmark utility specification gives

\[ \frac{1 - \tau_{h,2}}{1 - \tau_{h,1}} = \frac{1}{\varepsilon_2} \left( \frac{c_{1,t}}{c_{2,t+1}} \right)^{-\sigma_t} \left( \frac{h_{2,t+1}}{h_{1,t}} \right)^{\frac{1}{\sigma_2}}. \]  \( (63) \)

Combining equation 62 and 63 yields

\[ \frac{1 - \tau_{h,2}}{1 - \tau_{h,1}} = \frac{1}{\varepsilon_2} \left( \frac{\beta \rho_t}{\rho_{t+1} \theta} \right) \left( \frac{h_{2,t+1}}{h_{1,t}} \right)^{\frac{1}{\sigma_2}}. \]  \( (64) \)
The ratio of first order equations for the government with respect to young and old hours is
\[
\frac{\rho_t \beta}{\varepsilon_2 \rho_{t+1} \theta} \left( \frac{h_{2,t+1}}{h_{1,t}} \right) \frac{\lambda}{\sigma_2} = \frac{1 + \lambda_t (1 + \frac{1}{\sigma_1})}{1 + \lambda_t (1 + \frac{1}{\sigma_2})}.
\] (65)

Combining equation 65 and 64 generates the following expression for labor taxes
\[
\frac{1 - \tau_{h,2}}{1 - \tau_{h,1}} = \frac{1 + \lambda_t (1 + \frac{1}{\sigma_2})}{1 + \lambda_t (1 + \frac{1}{\sigma_1})} = 1.
\] (66)

**A.2 LBD**

The Lagrangian for this LBD specification is
\[
\mathcal{L} = c_{1,t}^{1-\sigma_1} - \frac{\chi}{1 - \sigma_1} - h_{1,t}^{1 + \frac{1}{\sigma_1}} + \beta c_{2,t+1}^{1-\sigma_1} - \frac{\chi}{1 - \sigma_1} h_{2,t+1}^{1 + \frac{1}{\sigma_2}}
\]
\[
- \rho_t (c_{1,t} + c_{2,t} + K_{t+1} - K_t + G_t - rK_t - w(h_{1,t} + h_{2,t} s_2))
\]
\[
- \rho_{t+1} \theta (c_{1,t+1} + c_{2,t+1} + K_{t+2} - K_{t+1} + G_{t+1} - rK_{t+1} - w(h_{1,t+1} + h_{2,t+1} s_2))
\]
\[
+ \lambda_t (c_{1,t}^{1-\sigma_1} + \beta c_{2,t+1}^{1-\sigma_1} - \chi h_{1,t}^{1 + \frac{1}{\sigma_2}} + \frac{\chi \beta h_{2,t+1} s_{h_1} (t+1)}{s_2} - \beta \chi h_{2,t+1}^2)
\] (67)

where \( \rho \) is the Lagrange multiplier on the resource constraint and \( \lambda \) is the Lagrange multiplier on the implementability constraint. The first order conditions with respect to labor, capital and consumption are
\[
\rho_t (1 + h_{2,t+1} s_{h_1} (t+1)) = \chi h_{1,t}^{1 + \frac{1}{\sigma_2}} (1 + \lambda_t (1 + \frac{1}{\sigma_2})) - \theta \rho_{t+1} h_{2,t+1} s_{h_1} (t+1)
\]
\[
+ \lambda_t \chi h_{2,t+1}^{1 + \frac{1}{\sigma_1}} \beta h_{1,t} \left[ \frac{s_{h_1} (t+1)^2}{s_2^2} - \frac{s_{h_2} h_2 (t+1)}{s_2} \right]
\] (68)
\[
\rho_{t+1} \theta s_2 = \beta \chi h_{2,t+1}^{1 + \frac{1}{\sigma_1}} \left[ 1 + \lambda_t (1 + \frac{1}{\sigma_2}) + (1 + \frac{1}{\sigma_2}) h_{1,t} s_{h_1} (t+1) \lambda_t \right]
\] (69)
\[
\rho_t = \theta (1 + r) \rho_{t+1}
\] (70)
\[
\rho_t = c_{1,t}^{1-\sigma_1} + \lambda_t (1 - \sigma_1) c_{1,t}^{-\sigma_1}
\] (71)
and
\[
\theta \rho_{t+1} = \beta c_{2,t+1}^{1-\sigma_1} + \beta \lambda_t (1 - \sigma_1) c_{2,t+1}^{-\sigma_1}.
\] (72)

The first order conditions with respect to capital and consumption are the same in the exogenous (59, 60, and 61) and LBD models (70, 71, and 72). Therefore equation 13 still holds for this model and therefore the optimal tax on capital is still zero when the government can condition labor income taxes on age.

Combining the first order equations for the governments problem with respect to capital and consumption yields
\[
\left( \frac{c_{2,t+1}}{c_{1,t}} \right)^{\sigma_1} = \frac{\beta \rho_t}{\rho_{t+1} \theta}
\] (73)

Taking the ratio of the agent’s first order conditions, equations 25 and 26 and combining with equation 73...
yields

$$\frac{1 - \tau_{h,1}}{1 - \tau_{h,2}} = \left( \frac{h_{1,t}}{h_{2,t+1}} \right) \frac{\sigma_t}{\beta} \left( \frac{\rho_{t+1} \theta s_2}{\beta \rho_t} \right) - \frac{h_{2,t+1} s h_2 (t + 1)}{1 + r (1 - \tau_k)}. \tag{74}$$

Combining equations 74, 68 and 69 the ratio of the optimal taxes on labor is,

$$\frac{1 - \tau_{h,1}}{1 - \tau_{h,2}} = \frac{\left[ 1 + h_{2,t+1} s h_2 (t + 1) \right] \left[ 1 + h_{1,t+1} \left( \frac{h_{1,t+1} s h_2 (t + 1)}{\sigma_t} \right) \right]}{1 + h_{2,t+1} s h_2 (t + 1) \sigma^2_{t+1} \left[ 1 + h_{1,t+1} \left( \frac{h_{1,t+1} s h_2 (t + 1)}{\sigma_t} \right) \right] - \frac{h_{2,t+1} s h_2 (t + 1)}{\sigma^2_{t+1}}. \tag{75}$$
B Competitive Equilibrium
For Online Publication

B.1 LBD Model

An agent’s state variables in the LBD model are assets \( a \), previous periods human capital \( s \), age \( j \), ability \( \alpha \), persistent shock \( \nu \), and idiosyncratic shock \( \theta \). For a given set of exogenous demographic parameters \( \{ n, \Psi_j \} \), a sequence of skill accumulations parameters \( \{ \Omega_j \} \), a government labor tax function \( T_l : \mathbb{R}_+ \to \mathbb{R}_+ \), a government capital tax function \( T_k : \mathbb{R}_+ \to \mathbb{R}_+ \), a social security tax rate \( \tau_{ss} \), social security benefits \( SS \), a production plan for the firm \( (N, K) \), a age-specific human capital accumulation function \( S : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+ \), a utility function \( U : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+ \), a stationary competitive equilibrium consists of agents’ decision rules \( \{ c, h \} \) for each state \( x \), factor prices \( \{ w, r \} \), transfers \( Tr \), and the distribution of individuals \( \{ \mu(x) \} \) such that the following holds:

1. Given prices, policies, transfers, benefits, and that \( \omega \) follows equation 37, the agent maximizes equation 35 subject to

\[
c + a' = w\omega h - \tau_{ss} w\omega h, + (1 + r)(a + Tr) - T_l[w\omega h(1 - .5\tau_{ss})] - T_k[r(a + Tr)],
\]

for \( j < j_r \), and

\[
c + a' = SS + (1 + r)(a + Tr) - T_k[r(a + Tr)],
\]

for \( j \geq j_r \).

Additionally,

\[
c \geq 0, 0 \leq h \leq 1, a \geq 0, a_1 = 0.
\]

2. Prices \( w \) and \( r \) satisfy

\[
r = \zeta \left( \frac{N}{K} \right)^{1-\zeta} - \delta
\]

and

\[
w = (1 - \zeta) \left( \frac{K}{N} \right)^\zeta.
\]

3. The social security policies satisfy

\[
SS = b \frac{wN}{\sum_{j<j_r} \mu(x)}
\]

and

\[
\tau_{ss} = \frac{\sum_{j>j_r} ss \mu(x)}{\sum_{j<j_r} w\omega \mu(x)}.
\]
4. Transfers are given by

\[ Tr = \sum (1 - \Psi) a' \mu(x). \] (83)

5. Government balances its budget

\[ G = \sum T^k [r(a + Tr)] \mu(x) + \sum_{j < j_1} T^l [\omega h (1 - 0.5 \tau_{ss})] \mu(x). \] (84)

6. The market clears

\[ K = \sum a \mu(x), \] (85)

\[ N = \sum h \omega \mu(x), \] (86)

and

\[ \sum c \mu(x) + \sum a' \mu(x) + G = K \zeta N^{1-\zeta} + (1 - \zeta)K. \] (87)

7. The distribution of \( \mu(x) \) is stationary, that is, the law of motion for the distribution of individuals over the state space satisfies \( \mu(x) = Q_\mu \mu(x) \), where \( Q_\mu \) is a one-period recursive operator on the distribution.
References


