

# Optimal cost reimbursement of health insurers to reduce risk selection

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## Abstract

In the absence of a perfect risk adjustment scheme, reimbursing health insurers' costs can reduce risk selection in community-rated health insurance markets. In this paper, we develop a model in which insurers determine the cost efficiency of health care and have incentives for risk selection. We derive the optimal cost reimbursement function which balances the incentives for cost efficiency and risk selection. For health cost data from a Swiss health insurer, we find that an optimal cost reimbursement scheme should reimburse costs only up to a threshold.

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**Conflicts of interest:** None.

**Ethical background:** Theory and empirical study using anonymised data, no approval by a committee necessary.

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# 1 Introduction

Risk selection is a major concern in community-rated health insurance markets. Insurers have an incentive to discriminate against high risks and to attract low risks in such markets since they are not allowed to charge risk-based premiums. To avoid risk selection, regulators frequently impose open enrollment, define standardized benefit packages and implement risk adjustment schemes. However, these measures may not reduce incentives for risk selection sufficiently. In this case, reimbursing health insurers' costs can be useful. Here regulators face a *selection-efficiency trade-off* [Newhouse (1996)]. Lower incentives for risk selection will be accompanied by less cost efficiency.

Although the possible benefits of cost reimbursement are generally recognized, there has been little theoretical work on the characteristics of an optimal cost reimbursement function. Usually cost reimbursement is regarded as a “mandatory reinsurance program with regulated reinsurance premiums” [van de Ven and Ellis (2000, p. 818)]. This analogy suggests that optimal cost reimbursement is similar to an optimal insurance contract. Here Arrow (1974) and Raviv (1979) have shown that full or partial coverage above a deductible is optimal. For optimal cost reimbursement this implies that costs should only be reimbursed above a threshold. This *outlier risk sharing* is used in practice. In Germany, for example, 60% of individual health care costs which exceed a threshold of €21,352 were reimbursed in 2008.

In this paper, we treat cost reimbursement as an *incentive problem*. We analyze the problem of a regulator who wants to find the optimal balance between incentives for cost efficiency and risk selection. On the one hand, insurers can influence medical costs by negotiating lower prices with providers and by checking medical bills carefully to limit unnecessary treatment or upcoding. To do so, they have to exert higher effort. On the other hand, insurers spend resources on risk selection. Therefore, health insurers have to cover three kinds of costs: medical costs, effort costs, and risk selection costs. The regulator's aim is to minimize the sum of these costs by the choice of the cost reimbursement function to keep health insurance premiums as low as possible.

We characterize the optimal cost reimbursement function for this setting and show that at the optimum the marginal reimbursement rate is either 0 or 100%. We

also find that it can be optimal to reimburse costs only up to a threshold, but not above, i.e., additional costs above the threshold do not lead to further cost reimbursement. This is the opposite of outlier risk sharing and shows that the theory of optimal insurance does not carry over to optimal cost reimbursement to reduce risk selection. Indeed, for a data set of health care costs from a Swiss health insurer, optimal cost reimbursement does not resemble outlier risk sharing, but reimburses costs only up to a threshold.

The paper is structured as follows. In Section 2, we discuss incentive schemes to reduce risk selection and place our analysis in the context of the literature. Section 3 presents the model. The optimal cost reimbursement formula is derived and discussed in Section 4. In Section 5, we present two examples and calculate optimal cost reimbursement formulas based on data from a Swiss health insurer. Section 6 summarizes the results and concludes.

## **2 Incentive schemes to reduce risk selection**

Most of the literature on incentive schemes to reduce risk selection in health insurance markets has so far focused on risk adjustment. Many empirical studies have examined the properties of possible risk adjusters [see van de Ven and Ellis (2000) for a survey]. Risk adjustment schemes equalize observable differences in average costs by basing transfer payments on predicted health care expenditure conditional on the values of the risk adjusters. Glazer and McGuire (2000, 2002) and Frank et al. (2000), however, show that this approach is not optimal if observable characteristics are only imperfect signals for individual health status. A risk adjustment scheme which takes this into account can be much more effective in reducing risk selection. A further proposal has been advanced by Barros (2003). He shows that an ex-post fund can in principle avoid risk selection without comprising on cost efficiency.

However, it remains unclear whether risk adjustment schemes or ex-post funds can sufficiently reduce risk selection. The main problem is the availability of data. Often, only few characteristics such as age and gender can easily be obtained. Even if further indicators, in particular diagnostic information, are used to improve current risk adjustment schemes, risk selection may still be highly profitable [Newhouse (1994)]. The same problem applies to ex-post funds. As Barros (2003, p. 437) points out, it must be possible to assign spending of insurers to specific dis-

eases. Without detailed diagnostic information, ex-post funds will therefore not be able to rule out risk selection. For this reason, there is need for second-best solutions which balance the selection-efficiency trade-off and reimbursing insurers' costs can be useful.<sup>1</sup>

Several forms of cost reimbursement have been proposed in the literature [see van de Ven and Ellis (2000), Section 4.1]. On the one hand, there are cost reimbursement schemes which apply to all individuals and are similar to reinsurance contracts. These include outlier risk sharing and proportional risk sharing which reimburses a fixed percentage of all costs. On the other hand, forms of cost reimbursement have been put forward which are limited to a specific group.<sup>2</sup> Van de Ven and van Vliet (1992) propose *risk sharing for high risks* which allows insurers to designate a specified percentage of their insured for which all health care costs will be reimbursed. *Risk sharing for high costs* is considered by van Barneveld et al. (2001). Under this scheme, all health care costs of a predetermined number of individuals with the highest actual costs are paid by the regulator.

In empirical studies, van Barneveld et al. (1998, 2001) compare these selective forms of cost reimbursement to outlier risk sharing and proportional risk sharing. They find that *risk sharing for high risks* as well as *risk sharing for high costs* is superior in reducing incentives for risk selection to outlier risk sharing and proportional risk sharing. These forms of risk sharing are more effective in reimbursing only the costs of high-risk types without sharing the costs of low-risk types.

In our analysis, we start from the assumption that the cost reimbursement scheme applies to all individuals. As opposed to the existing literature, we do not base our cost reimbursement function on reinsurance principles. Instead, we formulate a model and derive the optimal cost reimbursement function. Our result can be compared directly to other formulas which apply to all individuals, in particular to outlier risk sharing. We cannot say whether our approach is superior to the selective forms of cost reimbursement. In future studies of different cost reimbursement approaches, it would be interesting to compare risk sharing for high risks or high costs to our optimal cost reimbursement approach.

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<sup>1</sup>Marchand et al. (2003) show that prior expenditure can also be a useful risk adjuster to reduce risk selection.

<sup>2</sup>A further possibility is to make cost reimbursement dependent on a medical condition [see van de Ven and Ellis (2000, p. 822)].

## 3 The model

### 3.1 Basic assumptions

We analyze a perfectly competitive health insurance market in which the regulator wants to make medical services available to all individuals at a price independent of their risk type. This objective can be motivated by the principle of equal access [see Hurley (2000, p. 89–90)]. It can also be justified by the maximization of a social welfare function. In a general framework including health and productivity differences, Cremer and Pestieau (1996) have shown that uniform health insurance premiums are optimal if health and productivity are positively correlated.

The regulator implements the equity objective by community rating and open enrollment, i.e., he requires insurers to quote a uniform premium for all their insured and to accept all individuals applying for insurance. Furthermore, insurers offer a standardized health insurance package. Having taken care of equity concerns, the regulator's problem is to ensure an efficient provision of health insurance. He therefore must consider possible efficiency losses due to risk selection. Here, we assume that the regulator is able to enforce the provision of medical care at the desired quality. This rules out that insurers select risks by distorting the benefit package [see Selden (1998, 1999), Glazer and McGuire (2000) and Frank et al. (2000)]. Our focus is on direct risk selection where insurers can identify the risk type and spend resources to impose barriers for high-risk individuals and to attract low-risk individuals [Zweifel et al. (2009, p. 253)].<sup>3</sup>

From an efficiency perspective, a key concern is that direct risk selection generates additional expenses by insurers which, in the end, will have to be paid for by individuals. The regulator's objective is to reduce these costs to the extent that health insurance premiums remain as low as possible. He also considers the insurers' effort to control costs, e.g., by checking medical bills more carefully or by negotiating lower prices with providers. This effort may be reduced by actions of the regulator against risk selection. In the following, we assume that higher effort  $e$  will decrease costs to treat an illness at the expense of effort costs  $v(e)$  with

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<sup>3</sup>For example, they may process applications of high risks only slowly or 'advise' that another insurer is more appropriate for them. Low risks, on the other hand, may be captured by selective advertisement or by offering supplementary benefits at a discount [see Kifmann (2006)].

$v'(e) > 0$  and  $v''(e) \geq 0$ .<sup>4</sup> Costs to treat a patient depend on the effort level  $e$  and on the severity  $m$  of the patient's illness where  $0 \leq m \leq M$ . We assume

$$C(e, m) = c(e)m, \quad c(e) > 0, c'(e) < 0, c''(e) > 0. \quad (1)$$

This cost function particularly fits a situation in which insurers negotiate a baseline reimbursement factor with providers.<sup>5</sup>

An individual can be a high risk  $h$  or a low risk  $l$ . Expected costs of the high-risk type are larger than expected costs of the low-risk type. The proportion of  $l$ -types is  $\theta$ . For each risk type  $i \in \{l, h\}$ , severity  $m$  is distributed according to the distribution function  $F_i(m)$ . Since a substantial fraction of insured usually does not use any health services during a certain period we allow for  $F_i(0) > 0$ . We assume the distribution function to be continuously differentiable for all  $m \geq 0$  and label the respective density function  $f_i(m)$ .<sup>6</sup> For  $m > 0$  we have

$$F_i(m) = F_i(0) + \int_0^m f_i(s) ds.$$

Expected costs of each risk type given effort level  $e$  correspond to

$$E_i[C(e, m)] = \int_0^M c(e)m f_i(m) dm \quad \text{with} \quad E_h[C(e, m)] > E_l[C(e, m)]. \quad (2)$$

For an insurer, whose share of  $l$ -types among his insured is  $\theta$ , expected costs are

$$E[C(e, m)] = \theta E_l[C(e, m)] + (1 - \theta) E_h[C(e, m)]. \quad (3)$$

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<sup>4</sup>See Marchand et al. (2003) for a similar approach.

<sup>5</sup>The method we present in the following can also be applied to other cost functions. In footnote 15, we point out the implications for the optimal cost reimbursement formula.

<sup>6</sup>To be more precise, we assume the distribution function  $F_i(m)$  to be continuously differentiable for all  $m > 0$  and  $\lim_{m \rightarrow 0^+} F_i'(m)$  to exist; accordingly by  $f_i(0)$  we mean  $\lim_{m \rightarrow 0^+} f_i(m)$ .

### 3.2 The benchmark case: no cost reimbursement

Without cost reimbursement, a risk-neutral insurer with a representative share of the two risk types will choose the effort level  $\hat{e}$  that minimizes the sum of expected treatment costs and effort costs:

$$\hat{e} = \underset{e}{\operatorname{arg\,min}} \ E[C(e, m)] + v(e). \quad (4)$$

With this effort level, the lowest premium  $P$  the insurer can offer is

$$P = E[C(\hat{e}, m)] + v(\hat{e}). \quad (5)$$

If insurers were not able to practice risk selection, perfect competition between health insurers would lead to the same premium and individuals would choose their insurer randomly because they offer the same quality of the benefit package. Each insurer would end up with a representative share of the two risk types and the premium to cover expected medical costs and effort costs would be as stated in (5). Comparing this premium to expected costs of type  $i$  shows that insurers make an expected profit of

$$\begin{aligned} \pi_i &= E[C(e, m)] - v(e) - (E_i[C(e, m)] - v(e)) \\ &= E[C(e, m)] - E_i[C(e, m)] \end{aligned} \quad (6)$$

which is positive for an  $l$ -type, since  $E[C(e, m)] > E_l[C(e, m)]$ , and negative for an  $h$ -type.

These profits and losses create incentives for risk selection. Insurers will compete for earning profit  $\pi_l$  and for avoiding  $\pi_h$ . In the following, we assume that risk selection costs  $RSC$  are proportional to the absolute values of these profits, weighted by the share of the respective types, i.e.,

$$RSC = \alpha(\theta|\pi_l| + (1 - \theta)|\pi_h|) = \alpha(\theta\pi_l - (1 - \theta)\pi_h), \quad \alpha > 0. \quad (7)$$

Note that the term in bracket is equal to the mean absolute deviation of costs

$$\begin{aligned} MAD &= \theta \left| E_l[C(e, m)] - E[C(e, m)] \right| + (1 - \theta) \left| E_h[C(e, m)] - E[C(e, m)] \right| \\ &= \theta |\pi_l| + (1 - \theta) |\pi_h| \end{aligned} \quad (8)$$

which is used in applied research to measure the incentives for risk selection [see van Barneveld et al. (2000), Cummings et al. (2002)]. We therefore have  $RSC = \alpha MAD$ .

The parameter  $\alpha$  measures the extent of risk selection costs. Lorenz (2008) shows that these risk selection costs can be interpreted as the outcome of a rent-seeking contest between insurers with identical risk selection technology. In this contest, an insurer who invests more in risk selection than other insurers will have a higher share of  $l$ -types among his insureds than in the population. In equilibrium, however, all insurers expend the same resources on risk selection and end up with a representative share of the two risk types. With a standard Tullock contest success function [Tullock (1980)], the sum of all investments in risk selection is – as specified in (7) – a fraction of the sum of rents weighted by the share of the risk types.

Because insurers will end up with a representative share of risk types in equilibrium, they will choose effort  $\hat{e}$ . In the competitive equilibrium the premium will therefore be

$$P = E[C(\hat{e}, m)] + v(\hat{e}) + \alpha MAD \quad (9)$$

and will cover exactly treatment costs, effort costs and risk selection costs.<sup>7</sup>

### 3.3 Cost reimbursement

Given that risk selection costs drive up health insurance premiums, the regulator's efficiency concern is to reduce risk selection costs  $RSC = \alpha MAD$  to a degree that total average costs to organize and provide health care

$$AC(\tilde{e}) = E[C(\tilde{e}, m)] + v(\tilde{e}) + \alpha MAD \quad (10)$$

and therefore insurance premiums are minimized. Here  $\tilde{e}$ , the effort level chosen by insurers, may deviate from  $\hat{e}$ , the effort level in the absence of cost reimbursement, due to the measures taken by the regulator to reduce  $RSC$ .

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<sup>7</sup>Note that an insurer does not gain by refraining from risk selection: Although he saves on  $RSC$ , this is more than compensated by the increase in treatment costs due to a higher share of high risks; see Lorenz (2008).

If the regulator were able to observe the risk type, he could eliminate risk selection costs without affecting incentives for cost efficiency simply by collecting  $E[C(\hat{e}, m)]$  from the insurer for each insured and paying  $E_i[C(\hat{e}, m)]$  depending on the risk type  $i$ . Insurers set  $\tilde{e} = \hat{e}$  because incentives for cost efficiency are not distorted. With transfers equal to expected costs, expected profits for both risk types are equal to zero and incentives for risk selection are eliminated completely. Thus, the insurer collects the premium  $P$ , receives a net payment of  $E_h[C(\hat{e}, m)] - E[C(\hat{e}, m)]$  for each  $h$ -type from the regulator, and pays  $E[C(\hat{e}, m)] - E_l[C(\hat{e}, m)]$  to the regulator for each  $l$ -type. The regulator breaks even since

$$\theta (E[C(\hat{e}, m)] - E_l[C(\hat{e}, m)]) - (1 - \theta) (E_h[C(\hat{e}, m)] - E[C(\hat{e}, m)]) = 0. \quad (11)$$

In the following, we assume that the regulator cannot identify the risk type and is therefore not able to implement a perfect risk adjustment scheme. Neither can he observe  $e$  nor  $m$ . However, he observes treatment costs  $C$  and knows the distribution functions  $F_i(m)$  for each risk type.<sup>8</sup> This last assumption will turn out to be of key importance.

In this setting, transfers cannot depend on risk type, but only on treatment costs  $C$  and are captured by the reimbursement function  $r(C)$ . For an individual with cost  $C(e, m)$ , the health insurer receives  $r(C(e, m))$ . For those values of  $C$  that are mainly incurred by high risks, we should have  $r(C) > 0$ ; for those that are more likely incurred by low risks,  $r(C) < 0$  would be optimal. In this way, the regulator can indirectly pay transfers for high risks and receive transfers for low risks.<sup>9</sup>

We impose two restrictions on  $r(C)$ . The first one is that the regulator breaks even in expectation, i.e.,

$$E[r(C(\tilde{e}, m))] = \theta E_l[r(C(\tilde{e}, m))] + (1 - \theta) E_h[r(C(\tilde{e}, m))] = 0. \quad (12)$$

The second one is that the marginal reimbursement rate  $r'(C)$  may not be negative or larger than one. This rules out incentives for cost deflation and cost inflation:

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<sup>8</sup>The distribution function can be inferred from a representative sample with information about the risk type. See Section 5.2 for an illustration of this procedure.

<sup>9</sup>In practice, cost reimbursement schemes are frequently financed by a uniform flat rate and define a nonnegative cost reimbursement function. In such a framework,  $-r(0)$  corresponds to the uniform flat rate and  $r(C) - r(0)$  equals the cost reimbursement.

1.  $r'(C) \geq 0$

If  $r(C)$  is non-decreasing in  $C$ , then hiding costs cannot lead to higher profits for the insurer.

2.  $r'(C) \leq 1$

This restriction guarantees that the insurer cannot increase his profits by inflating costs.

With  $r'(C) > 0$  for at least one interval of  $C$ , we have marginal cost reimbursement which will reduce incentives for efficiency. Insurers will choose a lower effort level which will increase treatment costs.<sup>10</sup>

The regulator has to consider this distortionary effect of the reimbursement function: a reduction in risk selection costs  $RSC$  could be more than offset by a large increase in treatment costs which occurs if there is too much *marginal* cost reimbursement so that effort drops too much.

With cost reimbursement, the effort level chosen by an insurer after individuals have signed up is

$$\tilde{e} = \arg \min_e E[C(e, m) - r(C(e, m))] + v(e).$$

This determines treatment costs  $E[C(\tilde{e}, m)]$  and costs of effort  $v(\tilde{e})$ . Using (8) and (12), risk selection costs  $\alpha MAD$  depend on  $r(C)$  by

$$\begin{aligned} MAD = & (2\theta - 1)E[C(\tilde{e}, m)] + (1 - \theta)E_h[C(\tilde{e}, m) - r(C(\tilde{e}, m))] \\ & - \theta E_l[C(\tilde{e}, m) - r(C(\tilde{e}, m))]. \end{aligned} \quad (13)$$

Note that risk selection costs depend not only directly on  $r(C)$  but also indirectly because  $\tilde{e}$  is induced by  $r(C)$ .

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<sup>10</sup>The lower effort level, of course, also reduces effort costs; the sum of treatment and effort costs, however, increases.

Summarizing, we suppose the following sequence of events:

1. The regulator announces the reimbursement function  $r(C)$ .
2. Insurers set premiums and spend resources to attract low risks and to avoid high risks. Each insurer ends up with a representative share of the two risk types [see Lorenz (2008)]. Premiums are identical.
3. Insurers select organizational effort  $\tilde{e}$ .
4. During the insurance period, severity  $m$  and costs  $C(\tilde{e}, m)$  are determined.
5. At the end of the insurance period, the regulator reimburses  $r(C(\tilde{e}, m))$ .

When setting  $r(C)$  at stage 1, the regulator wants to minimize this premium and therefore total average costs. He faces the problem

$$\min_{r(C)} AC(\tilde{e}) = E[C(\tilde{e}, m)] + v(\tilde{e}) + \alpha MAD \quad (14)$$

subject to the following constraints:

balanced budget condition (12)

$r(C)$  induces  $\tilde{e}$

$0 \leq r'(C) \leq 1$ .

## 4 The optimal cost reimbursement formula

In this section, we derive the solution to the regulators optimization problem. It is analytically convenient to proceed in two steps. (1) For an arbitrary effort level  $\tilde{e}$ , we show how to select of all functions  $r(C)$ , which induce the insurer to choose  $\tilde{e}$ , the one that minimizes  $AC(\tilde{e})$ . These minimal total average costs with respect to  $r(C)$  are denoted by  $AC_{r_{opt}}(\tilde{e})$ . This first step already yields the main insights for the reimbursement function. (2) Of all possible effort levels  $\tilde{e}$ , the regulator chooses  $\tilde{e}^*$ , the one that minimizes  $AC_{r_{opt}}(\tilde{e})$ . The optimal function  $r(C)$  according to step (1) for  $\tilde{e}^*$  is then the globally optimal reimbursement function.

## 4.1 The optimal cost reimbursement function for a given effort level

When we fix the effort level  $\tilde{e}$ , the only term of the problem

$$\min_{r(C)} E[C(\tilde{e}, m)] + v(\tilde{e}) + \alpha MAD \quad (15)$$

that depends on  $r(C)$  is  $MAD$ . We can therefore simplify our optimization problem in step 1 to minimize the  $MAD$ . We proceed as follows.

- 1a) We determine the incentive constraint which guarantees that insurers choose effort level  $\tilde{e}$ .
- 1b) We reformulate our optimization problem as an optimal control problem with costs  $C$  as the integration variable.
- 1c) We solve the optimal control problem and characterize the optimal cost reimbursement function  $r(C)$ .

*Step 1a)* When insurers select effort  $e$  at stage 3 and face a cost reimbursement function  $r(C)$ , their optimization problem is to minimize medical and effort costs

$$\min_e \theta E_l [c(e)m - r(c(e)m)] + (1 - \theta) E_h [c(e)m - r(c(e)m)] + v(e).$$

The first-order condition is

$$\int_0^M [c'(e)m - r'(c(e)m)c'(e)m] [\theta f_l(m) + (1 - \theta)f_h(m)] dm + v'(e) = 0. \quad (16)$$

Rearranging terms yields

$$\int_0^M r'(c(e)m)m g(m) dm = k(e) \quad \text{with} \quad k(e) \equiv \int_0^M m g(m) dm + \frac{v'(e)}{c'(e)} \quad (17)$$

where  $g(m) \equiv \theta f_l(m) + (1 - \theta)f_h(m)$  is the average density function. A sufficient condition for the corresponding effort level to yield a global profit-maximum is

that the cost reimbursement function  $r(C)$  is concave.<sup>11</sup> From condition (17), it follows that the cost reimbursement function must satisfy

$$\int_0^M r'(c(\tilde{e})m)mg(m) dm = k(\tilde{e}) \quad (18)$$

if insurers are to choose effort level  $\tilde{e}$ . Condition (18) therefore defines the incentive constraint which guarantees that insurers choose  $\tilde{e}$ .

*Step 1b)* To derive the optimal cost reimbursement function  $r(C)$ , it is convenient to express our problem with  $C$  as the integration variable. We therefore transform the distribution functions  $F_i(m)$  and the density functions  $f_i(m)$  into functions of  $C$ . This yields  $\tilde{F}_i(C) = F_i(C/c(\tilde{e}))$  and  $\tilde{f}_i(C) = f_i(C/c(\tilde{e}))/c(\tilde{e})$  with support  $[0, c(\tilde{e})M] = [0, \bar{C}]$ .<sup>12</sup>

Expressing the *MAD*, that has to be minimized at this stage, in terms of  $C$  leads to

$$\begin{aligned} MAD = & (2\theta - 1)E[C(\tilde{e}, m)] + (1 - \theta) \left( -r(0)\tilde{F}_h(0) + \int_0^{\bar{C}} [C - r(C)]\tilde{f}_h(C) dC \right) \\ & - \theta \left( -r(0)\tilde{F}_l(0) + \int_0^{\bar{C}} [C - r(C)]\tilde{f}_l(C) dC \right). \end{aligned} \quad (19)$$

Skipping all the constant terms that do not depend on  $r(C)$  we arrive at the full optimization problem:<sup>13</sup>

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<sup>11</sup>If  $r(C)$  is a concave function, then the left hand side of (17) is non-decreasing in  $e$ . Since the function  $k(e)$  is a strictly decreasing function of  $e$ , the first-order condition must therefore characterize a global optimum. If  $r(C)$  is not concave, then it needs to be checked whether equation (17) guarantees an optimum.

<sup>12</sup>From  $C = c(\tilde{e})m$  it follows that  $m = C/c(\tilde{e})$ . The distribution functions in terms of  $C$  are therefore given by  $\tilde{F}_i(C) = F_i(C/c(\tilde{e}))$ . Differentiating with respect to  $C$  yields the corresponding density functions  $\tilde{f}_i(C) = f_i(C/c(\tilde{e}))/c(\tilde{e})$ .

<sup>13</sup>These terms are  $(2\theta - 1)E[C(\tilde{e}, m)]$ ,  $\int C\tilde{f}_l(C)$  and  $\int C\tilde{f}_h(C)$ .

$$\begin{aligned}
\min_{r(C)} \quad & r(0)[\theta\tilde{F}_l(0) - (1-\theta)\tilde{F}_h(0)] + \int_0^{\bar{C}} r(C)[\theta\tilde{f}_l(C) - (1-\theta)\tilde{f}_h(C)]dC \\
\text{s.t.} \quad & \int_0^{\bar{C}} r'(C)C\tilde{g}(C) dC = c(\tilde{e})k(\tilde{e}) \tag{20} \\
& r(0)\tilde{G}(0) + \int_0^{\bar{C}} r(C)\tilde{g}(C) dC = 0 \tag{21} \\
& 0 \leq r'(C) \leq 1 \tag{22} \\
& r(0), r(\bar{C}) \text{ free} \tag{23}
\end{aligned}$$

where  $\tilde{g}(C) = \theta\tilde{f}_l(C) + (1-\theta)\tilde{f}_h(C)$  and  $\tilde{G}(C) = \theta\tilde{F}_l(C) + (1-\theta)\tilde{F}_h(C)$ . The first constraint is the transformed incentive constraint (18) which ensures that insurers choose  $\tilde{e}$ . The second constraint corresponds to the balanced budget condition (12). The third constraint ensures that there are neither incentives for cost inflation nor cost deflation. Finally, (23) states that there are no restrictions with respect to the endpoints of  $r(C)$ .

*Step 1c)* The reformulated problem is an isoperimetric dynamic optimization problem due to the equality integral constraints.<sup>14</sup> It cannot be solved by setting up the Hamiltonian since we allow for  $F_l(0) > 0$ . In the Appendix, we therefore formulate the Lagrangian for the full problem. There, we derive the following result.

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<sup>14</sup>See Chiang (1992, p. 280) and Kamien and Schwartz (1991, p. 228).

**Proposition 1:** *The slope of the optimal cost reimbursement function is characterized by*

$$\left. \begin{array}{l} r'(C) = 1 \\ r'(C) \in [0, 1] \\ r'(C) = 0 \end{array} \right\} \text{for } 2\theta(1 - \theta)[\tilde{F}_l(C) - \tilde{F}_h(C)] - \bar{\eta}C\tilde{g}(C) \left\{ \begin{array}{l} > 0 \\ = 0 \\ < 0 \end{array} \right. \quad (24)$$

where  $\bar{\eta} > 0$  is the constant Lagrange-multiplier associated with the incentive constraint (18). Condition (24) and the zero-budget constraint

$$r(0)\tilde{G}(0) + \int_0^{\bar{c}} r(C)\tilde{g}(C) dC = 0$$

determine  $r(0)$ .

Condition (24) states that the optimal cost reimbursement function considers for every cost level  $C$  whether the reduction in incentives for risk selection outweighs the efficiency costs of cost reimbursement. The distribution functions  $\tilde{F}_i(C)$ , the joint density function  $\tilde{g}(C)$  and the Lagrange-multiplier  $\bar{\eta}$  are crucial for this decision. More insights can be obtained by decomposing condition (24) into two terms with a natural interpretation:

1. *The anti-selection term*  $2\theta(1 - \theta)[\tilde{F}_l(C) - \tilde{F}_h(C)]$

With the share of low-risk types  $\theta$  given, a large difference  $\tilde{F}_l(C) - \tilde{F}_h(C) = (1 - \tilde{F}_h(C)) - (1 - \tilde{F}_l(C))$  tends to favor cost reimbursement. To explain this effect, it is important to note that  $r'(C) = 1$  increases cost reimbursement for all individuals with costs *above*  $C$ . This follows from the restriction  $r'(C) \geq 0$ . Since  $1 - \tilde{F}_i(C)$  denotes the share of  $i$ -types with costs higher than  $C$ ,  $(1 - \tilde{F}_h(C)) > (1 - \tilde{F}_l(C))$  implies that there are relatively more  $h$ -types with costs above  $C$  than  $l$ -types. Increasing cost reimbursement at  $C$  therefore reimburses costs more for  $h$ -types than for  $l$ -types. This implies that the MAD must fall.

2. *The cost-efficiency term  $\bar{\eta}C\tilde{g}(C)$*

A large value of  $\bar{\eta}C\tilde{g}(C)$  calls for no cost reimbursement. This is because  $C\tilde{g}(C)$  corresponds to the share of total costs at  $C$ .<sup>15</sup> If this share is large, then cost reimbursement at  $C$  tends to have a large negative impact on the incentives for efficiency and therefore calls for no cost reimbursement. This effect is increasing in  $\bar{\eta}$ , the Lagrange-multiplier, which captures the importance of incentives for cost efficiency. A higher  $\tilde{e}$ , i.e., higher incentives for cost efficiency, increases  $\bar{\eta}$ . For a given value of the anti-selection term, a larger value of  $\bar{\eta}$  therefore implies less cost sharing.

Note that the slope of the optimal cost reimbursement function is indeterminate for

$$2\theta(1 - \theta)[\tilde{F}_l(C) - \tilde{F}_h(C)] = \bar{\eta}C\tilde{g}(C). \quad (25)$$

However, this equality usually only holds for single values of  $C$ : the function on the left-hand side of (25) usually only crosses the function on the right-hand side, but is not identical for some interval. Thus, the indeterminacy only occurs at single points and does not influence the shape of the reimbursement function.

Except for these single points, the slope of the optimal reimbursement function is either zero or one, but never in between. This can be explained in the following way: a slope of, for example, 0.5 for some interval  $[C_1, C_2]$  could be replaced by a slope of 1 for an interval half as large within  $[C_1, C_2]$  where  $\tilde{g}(C)$  is small and a slope of 0 for the other half where  $\tilde{g}(C)$  is large. This shifts the marginal cost reimbursement to those values of  $C$  where only few costs occur and is therefore superior with respect to cost efficiency.

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<sup>15</sup>For other cost functions than  $C(e, m) = c(e)m$  this part of the cost efficiency term is different. For example, if  $C(e, m) = c(e) + m$ , then the cost efficiency term is  $\bar{\eta}\tilde{g}(C)$ .

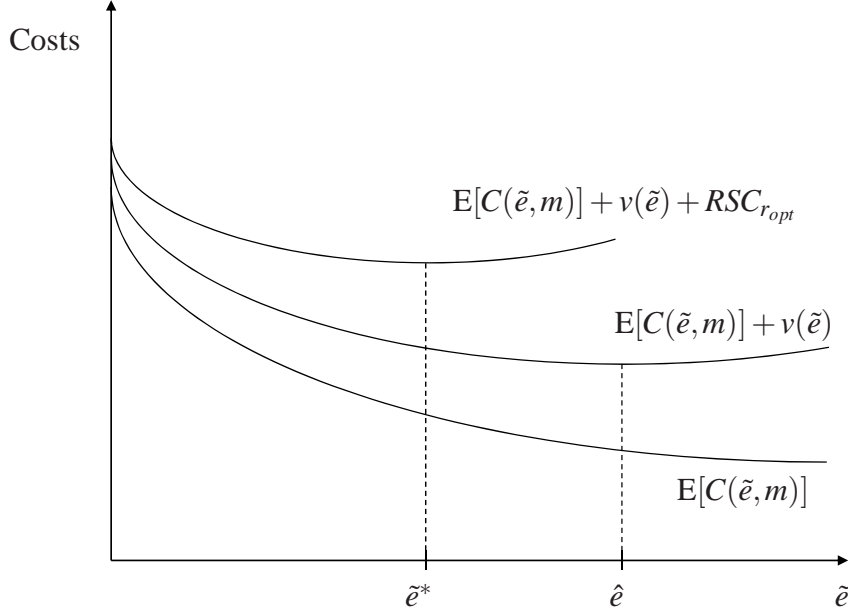


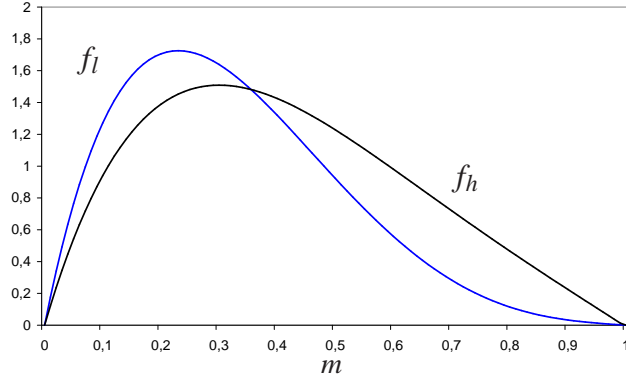
Figure 1: Minimized total average costs and the optimal effort level

## 4.2 The optimal effort level

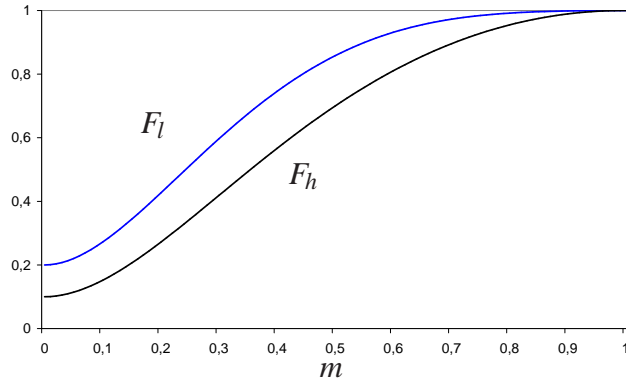
Having determined the optimal cost reimbursement function for a given level of effort  $\tilde{e}$ , in the second step the optimal level of  $\tilde{e}$  has to be determined. The procedure is illustrated in Figure 1. For each  $\tilde{e}$ , risk selection costs with the optimal reimbursement function  $r$  are  $RSC_{r_{opt}}$ ; total average costs are then

$$AC_{r_{opt}}(\tilde{e}) = E[C(\tilde{e}, m)] + v(\tilde{e}) + RSC_{r_{opt}}.$$

The optimal effort level  $\tilde{e}^*$  is given by the minimum of  $AC_{r_{opt}}(\tilde{e})$ . Figure 1 also shows the effort level  $\hat{e}$  which would be optimal in the absence of risk selection. At this level, however, the marginal benefit of reducing risk selection costs outweighs the efficiency losses due to cost reimbursement.



(a) Density functions



(b) Distribution functions

Figure 2: Example 1

## 5 Examples and empirical illustration

### 5.1 Examples

To illustrate our approach, we present two examples. We assume  $0 \leq m \leq 1$  and that there are two groups of equal size. The density and distribution functions for Example 1 are shown in Figure 2.<sup>16</sup> They are chosen so that they share three features we find in the data set analyzed in Subsection 5.2: both risk types have a positive probability of zero costs; for  $l$ -types, it is between 0.1 and 0.25 depending on age and gender, for  $h$ -types, the probability is always about half as large. The density for  $l$ -types is higher than the one for  $h$ -types for small values of  $m$  and

<sup>16</sup>The distribution functions are  $F_l(m) = 0.2 + 9.12m^2 - 20.96m^3 + 20.04m^4 - 8.88m^5 + 1.48m^6$  and  $F_h(m) = 0.1 + 6.25m^2 - 11.54m^3 + 9.5m^4 - 3.97m^5 + 0.66m^6$ .

	no cost reimbursement	optimal cost reimbursement	outlier risk sharing
$E[C(e, m)]$	0.315	0.335	0.335
$v(e)$	0.063	0.047	0.047
$RSC$	0.027	0.015	0.023
$AC$	0.405	0.396	0.404

Table 1: Costs in Example 1

lower for large values of  $m$ . Finally, the density approaches zero for the left and right boundary of  $m$ .

For the cost of effort function  $v(e)$  and the cost function  $c(e)$ , we assume

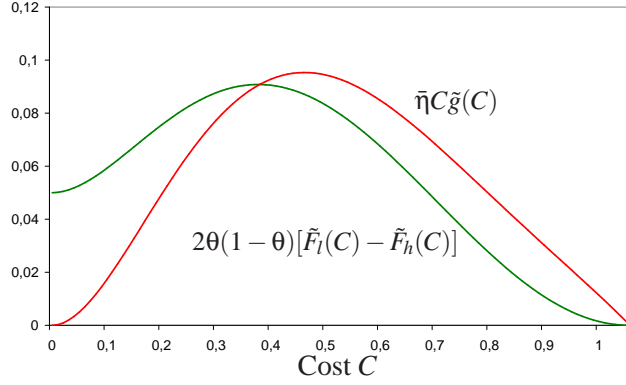
$$v(e) = e^{1/\gamma} \quad \text{and} \quad c(e) = \beta/e.$$

With these cost functions,  $\gamma$  determines the ratio of effort costs  $v(e)$  over medical costs  $E[C(e, m)]$ ; we set it equal to 20%.  $\beta$  is chosen such that  $c(\hat{e}) = 1$ . In the absence of cost reimbursement, average expected costs are  $E[C(\hat{e}, m)] = 0.315$ . The risk selection parameter  $\alpha$  is set to 0.5. Calculating the different cost components, total average costs without cost reimbursement are  $AC(\hat{e}) = 0.405$  (see Table 1).

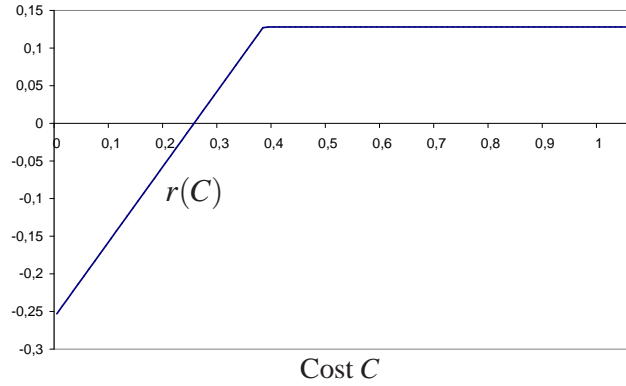
The globally optimal effort level  $\tilde{e}^*$  implies an increase of medical costs by 6.1%. Figure 3(a) shows the anti-selection term  $2\theta(1 - \theta)[\tilde{F}_l(C) - \tilde{F}_h(C)]$  and the cost efficiency term  $\tilde{\eta}C\tilde{g}(C)$  for this effort level. The functions intersect at  $C = 0.381$ . Where the anti-selection term is larger than the cost efficiency term, marginal cost reimbursement equals one, where it is below, there is no marginal cost reimbursement. Thus, costs should be reimbursed only up to a threshold. The balanced-budget condition requires  $r(0) = -0.253$ . This yields the optimal cost reimbursement function shown in Figure 3(b)

$$r(C) = \begin{cases} -0.253 + C & \text{for } C \leq 0.381 \\ 0.128 & \text{for } C > 0.381. \end{cases}$$

Expected costs  $E_l[C(\tilde{e}, m)]$  with optimal cost reimbursement are 0.335, an increase by 6.3% compared to no cost reimbursement (see Table 1). Considering effort costs  $v(e)$  as well, the increase is 1.1%. This is more than compensated by the decrease in risk selection costs by 45%. In total, average costs decline from 0.405 to 0.396, implying that cost reimbursement leads to a reduction in health insurance premiums by 2.2%.



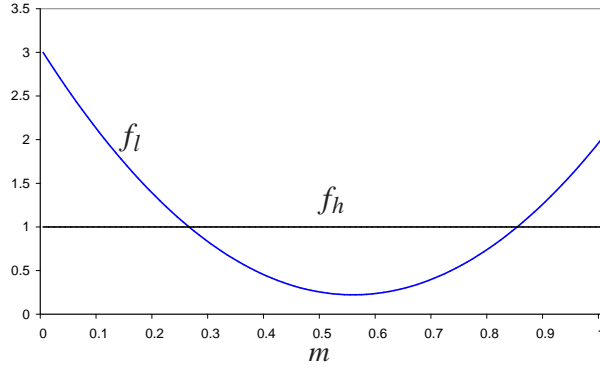
(a) Anti-selection and cost efficiency terms



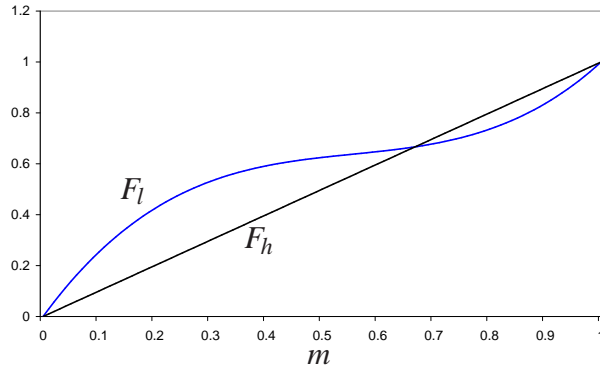
(b) The optimal cost reimbursement function

Figure 3: Example 1, optimal cost reimbursement

To compare our result to outlier risk sharing, we analyzed a scheme as in Germany in which 60% of all costs above a certain threshold are reimbursed. We set the threshold at  $C = 0.505$  such that insurers choose the same effort level as with our optimal cost reimbursement function. Our finding is that the reduction of risk selection costs hardly compensates for the increase in the other cost components. As shown in Table 1, average costs almost remain the same. The fact that outlier risk sharing does not consider the distribution of health care costs for each risk type is responsible for this performance. Outlier risk sharing does not exploit that the anti-selection term is larger than the cost-efficiency term in the interval  $[0, 0.381]$ . This can most clearly be seen at  $C = 0$  where the anti-selection term is positive because more  $l$ -types incur no costs than  $h$ -types and the cost-efficiency term is zero. Vice versa, outlier risk sharing reimburses costs for  $C \geq 0.505$  where



(a) Density functions



(b) Distribution functions

Figure 4: Example 2

the efficiency losses due to marginal cost reimbursement outweigh the gains due to lower risk selection costs. Only in the interval  $(0.381, 0.505]$ , outlier risk sharing is close to the optimal strategy. However, the marginal reimbursement rate is too low.

Example 2 gives an additional reason why outlier risk sharing can be harmful. Figure 4(a) shows that  $l$ -types are more likely not only to have low but also high illness severities. This leads to a crossing of the distribution functions in Figure 4(b).<sup>17</sup> This feature is present in some groups of the data set examined in the next subsection. This is not implausible.  $l$ -type individuals might suffer under acute illnesses with temporarily high costs.

<sup>17</sup>The distribution functions are  $F_l(m) = 3m^3 - 5m^2 + 3m$  and  $F_h(m) = m$ .

We concentrate on the change in the risk selection costs in this example. Optimal cost reimbursement with an increase of medical costs by 5.6% reduces risk selection costs by 85%. As in Example 1, a threshold is optimal above which no additional costs are reimbursed. By contrast, outlier risk sharing which reimburses 60% of all costs above a certain threshold and causes the same increase in medical costs leads to an *increase* in risk selection costs of 13.75%. Of this increase, 5.6% can be attributed to higher medical costs which increase the *MAD* in absence of cost reimbursement in the same proportion.<sup>18</sup> The additional 8.15%, however, are due the fact that outlier risk sharing reimburses costs at high cost levels where the anti-selection term  $2\theta(1 - \theta)[\tilde{F}_l(C) - \tilde{F}_h(C)]$  is *negative*. In this range, there are relatively more *l*-types. Reimbursing costs at these levels therefore mainly reimburses costs of *l*-types, making it even more attractive to practice risk selection. Thus, outlier risk sharing can actually be counterproductive with respect to reducing risk selection costs.

## 5.2 An empirical illustration

In the following, we show how our method can be applied to real data. We base our empirical analysis on administrative annualized data provided by a Swiss health insurer. The data set includes information on individual costs (all costs covered by the health insurer), hospitalization (whether an individual was treated in a hospital in a year), number of months insured, death and extent of coinsurance for the years 1997 to 1999 with 475,506 observations. We used the observations of 104,420 adult individuals insured in the years 1998 and 1999. Their average health care expenditure was CHF 3,250 (€ 2,025) in 1999.

Since we do not have information about insurer's risk selection activities and costs, we need to formulate a hypothesis about how insurers risk select. Our risk selection hypothesis is that health insurers can observe whether an individual was treated in a hospital in 1998. The group *h* is therefore given by those treated in a hospital in 1998. The *l*-types are the remaining individuals. We assume that the regulator is not able to obtain information on hospitalization.<sup>19</sup>

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<sup>18</sup>Since all costs, i.e.,  $E_l[C]$ ,  $E_h[C]$  and  $E[C]$ , increase by 5.6%, so does the *MAD*, the weighted difference between these costs.

<sup>19</sup>In practice, regulators should be able to obtain this information. However, so far it is not used

Table 2 on page 28 shows the percentage of individuals hospitalized in 1998 for each of the 30 age-gender-cells of the Swiss risk adjustment scheme. The fifteen age groups comprise 5 years, except for the youngest and oldest group. On average, 15.6% of the individuals were treated in a hospital in 1998. The treatment costs of these individuals were more than three times higher than the costs of those who were not hospitalized. The average cost ratio is highest for men, aged 31 to 35 years and tends to decrease with age. By contrast, the hospitalization rates increase with age, the exception being women who show a temporary decrease after their childbearing age. Overall, hospitalization is a high cost indicator for all groups, making it attractive for insurers to discriminate against those who were treated in a hospital.

To illustrate possible shapes of the optimal cost reimbursement function, we apply our method to each age-gender-cell of the Swiss risk adjustment scheme. Since no empirical estimate is available for the parameter  $\alpha$  which measures the extent of risk selection, we concentrate on the first step of our two-step-procedure and determine the optimal cost reimbursement function for a fixed level of effort  $\tilde{e}$  leading to an increase in medical costs by  $\tilde{x}\%$ . This allows us to compare our optimal cost reimbursement function to outlier risk sharing with respect to the functional form and the effectiveness of reducing risk selection costs for a given increase in medical costs. To make the results comparable for the 30 risk adjustment cells, we set the effort level such that medical costs increase by 1% in each cell.

To determine the optimal cost reimbursement function, we proceed as follows.

1. We use the same functions  $v(e)$  and  $c(e)$  as in our example and set  $\beta$  so that  $c(\hat{e}) = 1$ . The costs from our data set correspond to the costs if insurers choose this effort level since there is no cost reimbursement in Switzerland. Thus, we obtain  $C^{\text{act.}} = c(\hat{e})m = m$  and use actual costs  $C^{\text{act.}}$  to estimate the distribution functions  $F_i(m)$  for each group.

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in the Swiss risk adjustment scheme. We make this assumption mainly because hospital stays are included in our data set and we can therefore use this information to illustrate our method. Nevertheless, our results may be interesting for a regulator who does not want to use information on hospital stays in a risk adjustment scheme to avoid that insurers encourage excessive hospitalization.

2. Second, we derive the distribution functions for an increase in costs by 1% and apply our method.

We estimate the distribution function of actual costs  $C^{\text{act.}}$  in 1999 for the two groups nonparametrically.<sup>20</sup> Since there was a considerable share of observations with zero costs, we set  $F_i(0)$  equal to this share and determined  $f_i(C^{\text{act.}})$  with  $C^{\text{act.}} > 0$  by kernel density estimation.<sup>21</sup>

Applying our method, we find that the optimal cost reimbursement function generally has a slope of one for  $C = 0$  and for very small costs.<sup>22</sup> Here, the anti-selection term is positive because more  $l$ -types incur no costs than  $h$ -types and the cost-efficiency term is small as in Example 1 in the previous subsection. This is followed by an interval with zero marginal cost reimbursement of variable length after which full marginal cost reimbursement is optimal again. Then no marginal cost reimbursement is optimal for higher costs, or there are further intervals where the slope switches from zero to one and vice versa. There is always a cost level, however, above which the slope of the cost reimbursement function is zero. Thus, a threshold is optimal above which no additional costs are reimbursed as in our examples.

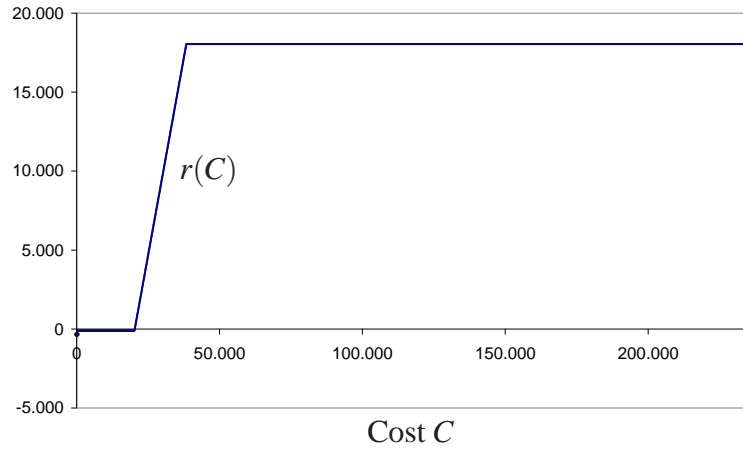
Figure 5 shows three typical results. In Figure 5(a), after a very small range with full cost reimbursement, no marginal cost reimbursement for cost below a threshold of about CHF 20,000 and above a threshold of about CHF 40,000 is optimal. Between CHF 20,000 to CHF 40,000, marginal cost reimbursement equals one. Figure 5(b) displays the case in which there is another interval in which full cost

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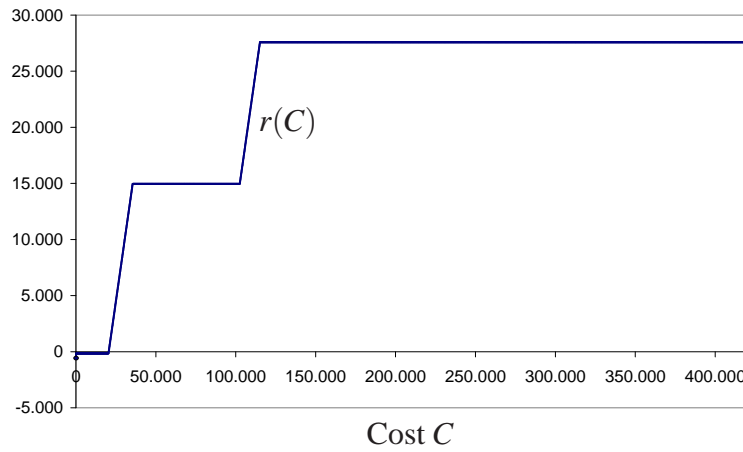
<sup>20</sup>A potential problem is overfitting with respect to time: since the regulator has to announce the reimbursement function at the beginning of period  $t$ , he can only use data from period  $t - 1$  to estimate the distribution functions. If these changed considerably over time, there would be overfitting by the method we apply. We checked this in our data set and found the distribution functions to be stable over time.

<sup>21</sup>When we chose a constant bandwidth for the kernel, we found that  $f_i(C^{\text{act.}}) = 0$  for a number of intervals for  $C^{\text{act.}} > \text{CHF } 10,000$ . This artificially improved our results because  $r'(C) = 1$  does not reduce incentives for efficiency at all whenever  $f_i(C) = 0$  for both groups. Therefore we transformed the data using a concave function. With the function  $\ln(C^{\text{act.}})$ , there are no intervals with  $f_i(C^{\text{act.}}) = 0$ . From the estimated distribution function  $\hat{F}_i(\ln(C^{\text{act.}}))$  we derive the distribution function  $F_i(C^{\text{act.}})$  and the density function  $f_i(C^{\text{act.}})$ . Alternatively, one could use a variable bandwidth and determine it endogenously by crossvalidation. This approach, however, requires very long computing time.

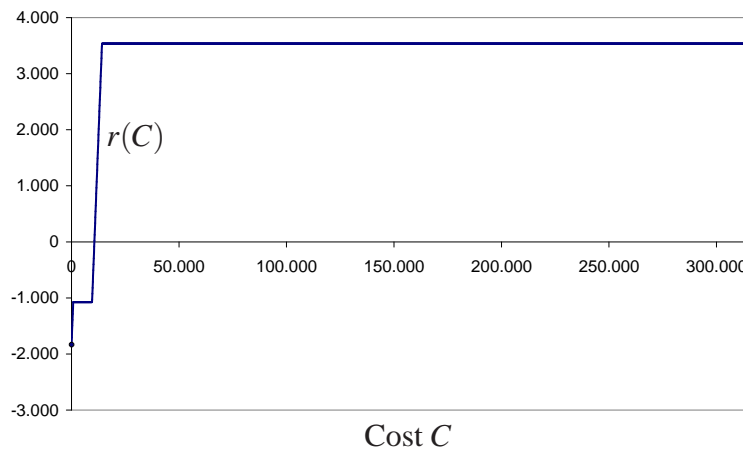
<sup>22</sup>The health insurers' second-order conditions were satisfied for all cost reimbursement functions.



(a) Cell F3 (women 31–35 yrs.)



(b) Cell F7 (women 51–55 yrs.)



(c) Cell M13 (men 81–85 yrs.)

Figure 5: Optimal  $r(C)$ -functions

reimbursement is optimal. The threshold above which no additional costs should be reimbursed is CHF 115,000. As before, there is no marginal cost sharing for low costs except for costs close to zero. Finally, Figure 5(c) shows a typical finding for elderly insured who are characterized by high levels of hospitalization. For these insured, the high risks (those who have been hospitalized) have a very low probability of incurring zero costs, so  $F_h(0) \approx 0$ . This leads to a high difference  $F_l(0) - F_h(0)$ , which calls for a rather high negative value of  $r(0)$ . The joint density function for high costs is comparatively high, implying a low threshold above which there is no marginal cost reimbursement. In this case, it is about CHF 14,000. There is also an interval of about CHF 8,000 with zero marginal cost sharing.

Table 2 contains an overview of our results and compares them to an outlier risk sharing scheme as in Germany which reimburses 60% of costs above a threshold. The threshold was chosen such that medical costs increase by 1%. It ranges from CHF 28,700 to CHF 114,900. We show the percentage reduction of risk selection costs (or *MAD*) for both optimal cost reimbursement and outlier risk sharing. On average, optimal cost reimbursement is 2.5 times more effective than outlier risk sharing in reducing risk selection costs (10.30% vs. 4.12%).

We checked whether our results are robust with respect to different degrees of marginal cost reimbursement for outlier risk sharing. We found that outlier risk sharing performs worse if this rate is increased to 80%. Risk selection costs fall only by 3.88%. Smaller rates slightly improve the efficiency of outlier risk sharing. For marginal cost reimbursement of 40% above a threshold, risk selection costs were reduced by 4.58%. Finally, we considered different increases in medical costs. For a 0.5% increase in medical costs, optimal cost sharing reduced risk selection costs by 5.54%, outlier risk sharing by 1.84%. For a 5% increase, the respective values are 38.26% and 23.14%. Again, optimal cost reimbursement performs much better.

Figure 6 displays the reduction in risk selection costs with optimal cost reimbursement and outlier risk sharing for a 1% increase in medical costs for all cells. These were arranged by the magnitude of the reduction of risk selection costs of outlier risk sharing. The first cell is the one where it is the least successful: here risk selection costs increase by 5.04%. The last cell is the one where it is the most

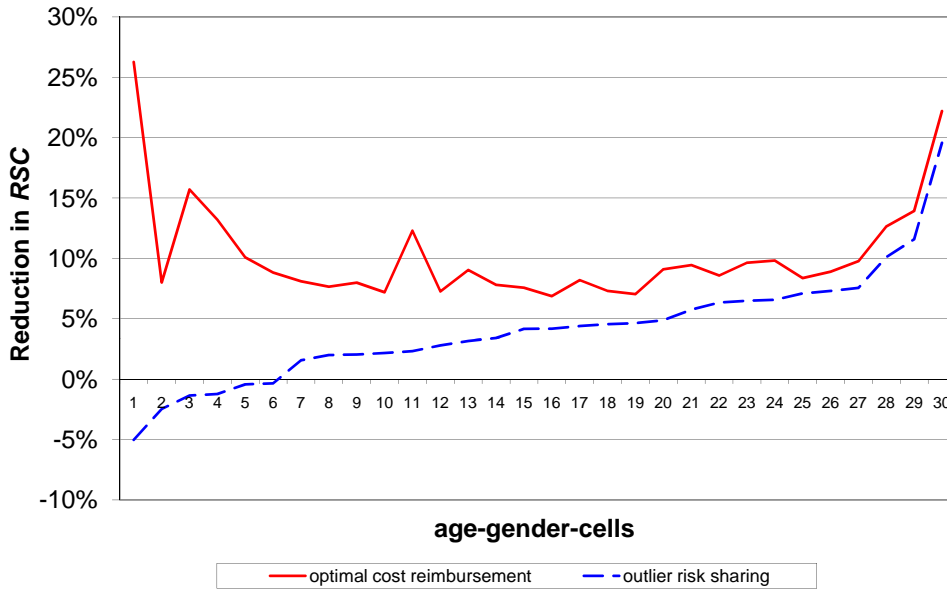


Figure 6: Optimal cost reimbursement and outlier risk sharing compared

successful and reduces risk selection costs by 19.57%. In 6 of 30 age-gender-cells risk selection costs increase under outlier risk sharing; in 4 of these cells this increase is higher than 1%, which shows that outlier risk sharing reimburses costs mostly for the  $l$ -types; these are the cells where the distribution functions cross for high values of  $C$  as in Example 2.

From Figure 6, it can be seen that optimal cost reimbursement performs well when outlier risk sharing works well. This comes at no surprise because optimal cost reimbursement can always mimic outlier risk sharing. However, it performs also well when outlier risk sharing does very badly. In our first example in the previous subsection, we showed why this is the case. Outlier risk sharing does not exploit the fact that differences in the distribution functions of the two risk types call for marginal cost reimbursement at lower levels and not for high costs. This effect is particular strong in cases which call for a low threshold above which now additional costs should be reimbursed as in Figure 5(c). Here, our approach reduces risk selection costs by 8.83% while outlier risk sharing increases these costs by 0.35%. For the oldest female group, risk selection costs can even be 31% lower if optimal cost reimbursement is used instead of outlier risk sharing.

## 6 Conclusion

The aim of this paper was to determine the optimal cost reimbursement of health insurers to reduce risk selection. We developed a model in which insurers influence the cost of health care with their organizational activities and where risk selection activities are increasing in the mean absolute deviation of costs. The optimal cost reimbursement function balances the incentives for cost efficiency and risk selection. For every cost level, it considers whether the reduction in incentives for risk selection outweighs the efficiency costs of cost reimbursement. The distribution of health care costs for each risk type is crucial. If there are relatively more high risk types with larger costs than low risk types, then incentives for risk selection can be reduced by marginal cost reimbursement. This has to be weighted against possible cost-efficiency losses. Here, the joint density function is of key importance. Comparing these two effects calls for a marginal cost reimbursement rate of either 0 or 100%.

When we applied our method to Swiss health cost data, we observed that costs should generally be reimbursed only up to a threshold. This is opposed to outlier risk sharing which is advocated in the literature and has been used in Germany. Our optimal cost reimbursement formula was also much more effective than outlier risk sharing. For a one percent increase in medical costs, we found that the mean decrease in risk selection costs is two and a half times larger if optimal cost reimbursement is used instead of outlier risk sharing. The fact that outlier risk sharing imposes a structure on cost reimbursement independent of the distribution of health care costs for each risk type is responsible for this difference. This shows that applying principles of reinsurance may not be appropriate if the objective is to reduce risk selection.

Our analysis was based on two risk types. It would be interesting to extend the analysis to multiple risk types. Also other functional forms of the cost function could be considered. Furthermore, we focused on organizational effort of insurers which affects the cost of all patients. Future work could examine the implications of patient-specific effort. Finally, our approach can be compared to selective forms of cost reimbursement such as risk sharing for high risks or high costs to our optimal cost reimbursement approach.

Cell	Age-Gender-Bracket	Number of observations	Percentage Hospitalized in 1998	Average Cost Hospitalized	Average Cost Non-Hosp.	Average Cost Ratio	Reduction of RSC in % Outlier Risk-sharing	Reduction of RSC in % Optimal Cost reimbursement
F1	Women 18–25	5,902	6.2	2,904	892	3.25	19.57	22.20
F2	Women 26–30	4,289	11.6	2,425	1,267	1.91	2.32	12.29
F3	Women 31–35	5,372	14.9	2,370	1,359	1.74	5.76	9.44
F4	Women 36–40	5,678	10.9	2,652	1,288	2.06	4.40	8.21
F5	Women 41–45	5,262	8.7	3,865	1,328	2.91	7.56	9.76
F6	Women 46–50	5,103	8.5	3,846	1,424	2.70	11.58	13.94
F7	Women 51–55	5,049	8.7	4,409	1,646	2.68	6.57	9.82
F8	Women 56–60	4,592	9.6	4,518	1,741	2.59	7.31	8.90
F9	Women 61–65	3,907	11.5	4,771	2,038	2.34	4.55	7.31
F10	Women 66–70	3,536	14.8	6,418	2,406	2.67	6.49	9.64
F11	Women 71–75	3,224	18.1	7,178	2,888	2.49	4.64	7.04
F12	Women 76–80	2,871	21.9	8,441	3,612	2.34	2.04	8.00
F13	Women 81–85	2,022	27.5	10,309	4,204	2.45	-0.44	10.09
F14	Women 86–90	1,564	35.9	12,157	5,738	2.12	-1.23	13.211
F15	Women 91+	803	48.9	13,872	8,025	1.73	-5.04	26.27
M1	Men 18–25	5,738	3.6	3,814	551	6.91	10.10	12.64
M2	Men 26–30	3,730	4.9	3,360	689	4.88	2.00	7.66
M3	Men 31–35	4,609	4.3	7,199	756	9.52	4.87	9.10
M4	Men 36–40	4,697	5.8	4,756	893	5.32	4.18	6.88
M5	Men 41–45	4,363	5.6	4,277	968	4.42	4.16	7.583
M6	Men 46–50	4,109	6.7	4,135	1,055	3.92	3.41	7.81
M7	Men 51–55	4,240	7.8	5,325	1,248	4.27	7.10	8.37
M8	Men 56–60	3,769	9.4	4,671	1,664	2.81	1.56	8.11
M9	Men 61–65	2,920	10.8	6,413	2,065	3.11	3.15	9.04
M10	Men 66–70	2,351	15.1	5,863	2,653	2.21	-2.47	8.00
M11	Men 71–75	1,852	19.1	7,922	3,498	2.26	6.34	8.58
M12	Men 76–80	1,389	23.0	7,239	3,764	1.92	2.16	7.20
M13	Men 81–85	805	26.8	9,372	4,844	1.93	-0.35	8.83
M14	Men 86–90	489	30.9	12,215	5,795	2.11	2.79	7.26
M15	Men 91+	185	37.3	13,847	8,011	1.73	-1.35	15.70
	Average	3,481	15.6	10,190	4,188	3.11	4.12	10.30

Table 2: Age-gender cells and results for a 1% increase in costs

## Appendix

Since we allow for  $F_i(0) > 0$  it is not possible to solve our isoperimetric dynamic optimization problem with free starting and end points by setting up the Hamiltonian. In the following, we therefore solve the complete problem. To save on notation, we define  $\tilde{H}(C) = (1 - \theta)\tilde{F}_h(C) - \theta\tilde{F}_l(C)$  and  $\tilde{h}(C) = (1 - \theta)\tilde{f}_h(C) - \theta\tilde{f}_l(C)$ . Then problem (20) is equivalent to the maximization problem

$$\max_{r(\cdot)} r(0)\tilde{H}(0) + \int_0^{\bar{c}} r(C)\tilde{h}(C) dC \quad (\text{A.1})$$

s.t.

$$r(0)\tilde{G}(0) + \int_0^{\bar{c}} r(C)\tilde{g}(C) dC = 0 \quad (\text{A.2})$$

$$\int_0^{\bar{c}} r'(C)C\tilde{g}(C) dC = k(\tilde{e})c(\tilde{e}) \quad (\text{A.3})$$

$$0 \leq r'(C) \leq 1 \quad (\text{A.4})$$

$$r(0), r(\bar{c}) \text{ free} \quad (\text{A.5})$$

$$(\text{A.6})$$

Now replace constraint (A.2) by

$$K(C) = \int_0^C r(s)\tilde{g}(s) ds \text{ with } K'(C) = r(C)\tilde{g}(C), K(0) = 0 \text{ and } K(\bar{c}) = -r(0)\tilde{G}(0).$$

Furthermore set  $r'(C) = u(C)$  and replace (A.3) by

$$L(C) = \int_0^C u(s)s\tilde{g}(s) ds \text{ with } L'(C) = u(C)C\tilde{g}(C), L(0) = 0 \text{ and } L(\bar{c}) = c(\tilde{e})k(\tilde{e}).$$

Therefore the problem is

$$\max_{r(\cdot)} r(0)\tilde{H}(0) + \int_0^{\bar{c}} r(C)\tilde{h}(C) dC \quad (\text{A.7})$$

subject to

$$\begin{aligned}
K'(C) &= r(C)\tilde{g}(C), & K(0) &= 0, & K(\bar{C}) &= -r(0)\tilde{G}(0) \\
L'(C) &= u(C)C\tilde{g}(C), & L(0) &= 0, & L(\bar{C}) &= k(\tilde{e})c(\tilde{e}) \\
r(0), r(\bar{C}) & \text{ free} \\
0 &\leq r'(C) \leq 1
\end{aligned}$$

We can now set up the Lagrangian

$$\begin{aligned}
\mathcal{L} &= r(0)\tilde{H}(0) + \int_0^{\bar{C}} \left\{ r(C)\tilde{h}(C) + \lambda(C)[u(C) - r'(C)] \right. & (A.8) \\
&\quad \left. + \mu(C)[K'(C) - r(C)\tilde{g}(C)] + \eta(C)[L'(C) - u(C)C\tilde{g}(C)] \right\} dC \\
&\quad + \gamma_1 K(0) + \gamma_2 [-r(0)\tilde{G}(0) - K(\bar{C})] + \gamma_3 L(0) + \gamma_4 [k(\tilde{e})c(\tilde{e}) - L(\bar{C})].
\end{aligned}$$

Note that  $\eta(C)$  is the Lagrange multiplier associated with the incentive constraint (A.3). Integrating  $\lambda(C)r'(C)$ ,  $\mu(C)K'(C)$  and  $\eta(C)L'(C)$  by parts we obtain

$$\begin{aligned}
\mathcal{L} &= r(0)\tilde{H}(0) + \int_0^{\bar{C}} \left\{ r(C)\tilde{h}(C) + \lambda(C)u(C) + \lambda'(C)r(C) \right. & (A.9) \\
&\quad \left. - \mu'(C)K(C) - \mu(C)r(C)\tilde{g}(C) - \eta'(C)L(C) - \eta(C)u(C)C\tilde{g}(C) \right\} dC \\
&\quad - [\lambda(\bar{C})r(\bar{C}) - \lambda(0)r(0)] + [\mu(\bar{C})K(\bar{C}) - \mu(0)K(0)] \\
&\quad + [\eta(\bar{C})L(\bar{C}) - \eta(0)L(0)] \\
&\quad + \gamma_1 K(0) + \gamma_2 [-r(0)\tilde{G}(0) - K(\bar{C})] + \gamma_3 L(0) + \gamma_4 [k(\tilde{e})c(\tilde{e}) - L(\bar{C})].
\end{aligned}$$

The first differential is

$$\begin{aligned}
\Delta\mathcal{L} &= \int_0^{\bar{C}} \left\{ [\tilde{h}(C) + \lambda'(C) - \mu(C)\tilde{g}(C)]\Delta r(C) \right. & (A.10) \\
&\quad \left. + [\lambda(C) - \eta(C)C\tilde{g}(C)]\Delta u(C) - \mu'(C)\Delta K(C) - \eta'(C)\Delta L(C) \right\} dC \\
&\quad + [\tilde{H}(0) + \lambda(0) - \gamma_2\tilde{G}(0)]\Delta r(0) - \lambda(\bar{C})\Delta r(\bar{C}) \\
&\quad + [-\mu(0) + \gamma_1]\Delta K(0) + [\mu(\bar{C}) - \gamma_2]\Delta K(\bar{C}) \\
&\quad + [-\eta(0) + \gamma_3]\Delta L(0) + [\eta(\bar{C}) - \gamma_4]\Delta L(\bar{C}) \\
&\quad + K(0)\Delta\gamma_1 + [-K(\bar{C}) - r(0)\tilde{G}(0)]\Delta\gamma_2 + L(0)\Delta\gamma_3 + [k(\tilde{e})c(\tilde{e}) - L(\bar{C})]\Delta\gamma_4
\end{aligned}$$

This yields the following conditions for optimality

$$\tilde{h}(C) + \lambda'(C) - \mu(C)\tilde{g}(C) = 0 \quad (\text{A.11})$$

$$\lambda(C) - \eta(C)C\tilde{g}(C) \begin{cases} > 0 & \Rightarrow & u(C) = 1 \\ = 0 & \Rightarrow & 0 \leq u(C) \leq 1 \\ < 0 & \Rightarrow & u(C) = 0 \end{cases} \quad (\text{A.12})$$

$$-\mu'(C) = 0 \quad \text{which implies} \quad \mu(C) = \bar{\mu} \quad (\text{A.13})$$

$$-\eta'(C) = 0 \quad \text{which implies} \quad \eta(C) = \bar{\eta} \quad (\text{A.14})$$

$$\lambda(0) = -\tilde{H}(0) + \gamma_2\tilde{G}(0) \quad (\text{A.15})$$

$$\lambda(\bar{C}) = 0 \quad (\text{A.16})$$

$$\gamma_2 = \mu(\bar{C}) \quad \text{which implies} \quad \gamma_2 = \bar{\mu}. \quad (\text{A.17})$$

$$K(0) = 0 \quad (\text{A.18})$$

$$r(0)\tilde{G}(0) + K(\bar{C}) = 0 \quad (\text{A.19})$$

$$L(0) = 0 \quad (\text{A.20})$$

$$L(\bar{C}) = k(\tilde{e})c(\tilde{e}). \quad (\text{A.21})$$

Integrating (A.11) yields

$$\begin{aligned} \lambda(C) &= \lambda(0) + \int_0^C \lambda'(s) \, ds \\ &= -\tilde{H}(0) + \bar{\mu}\tilde{G}(0) + \int_0^C -\tilde{h}(s) + \bar{\mu}\tilde{g}(s) \, ds \\ &= -\tilde{H}(C) + \bar{\mu}\tilde{G}(C). \end{aligned} \quad (\text{A.22})$$

With

$$0 = \lambda(\bar{C}) = -\tilde{H}(\bar{C}) + \bar{\mu}\tilde{G}(\bar{C}) = 0$$

we get  $\bar{\mu} = (1 - 2\theta)$  which simplifies (A.22) to

$$\lambda(C) = -\tilde{H}(C) + (1 - 2\theta)\tilde{G}(C) = 2\theta(1 - \theta)[\tilde{F}_i(C) - \tilde{F}_h(C)].$$

Inserting into (A.12) we obtain

$$2\theta(1-\theta)[\tilde{F}_l(C) - \tilde{F}_h(C)] - \bar{\eta}C\tilde{g}(C) \begin{cases} > 0 & \Rightarrow u(C) = r'(\bar{C}) = 1 \\ = 0 & \Rightarrow 0 \leq u(C) = r'(\bar{C}) \leq 1 \\ < 0 & \Rightarrow u(C) = r'(\bar{C}) = 0 \end{cases} . \quad (\text{A.23})$$

which is equivalent to condition (24). Now  $\bar{\eta}$  needs to be chosen such that (A.21) is satisfied. This guarantees

$$\int_0^{\bar{C}} r'(C)C\tilde{g}(C) dC = k(\tilde{e})c(\tilde{e}).$$

Finally  $r(0)$  is set such that (A.19) is satisfied which implies

$$r(0)\tilde{G}(0) + \int_0^{\bar{C}} r(C)\tilde{g}(C) dC = 0. \quad (\text{A.24})$$

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