Estimating gender specific sibling interaction effects

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Abstract

Building on sibling difference regressions, a new IV strategy for estimating heterogenous interaction effects in the linear peer effects model for siblings is proposed. Instruments are constructed from individual covariates that are correlated with the unobserved common family factor. Identification depends on a testable restriction, standardized direct responses to variations in own covariates differ across different types of siblings. The framework is used to estimate sibling interaction effects in long run wages by gender in the 1979 NLSY dataset.
Many children, parents and teachers believe and act upon the belief that siblings affect each other in their behavior. These effects may differ by a child’s gender and the gender of the sibling. However it has been difficult for empirical researchers to identify and quantify these sibling interaction effects with observational data. Since Manski 1993, researchers know that the correlation in siblings’ behaviors is due to interaction effects as well as observed and unobserved factors common to the family. The unobserved common family effects make it difficult to identify the interaction effects without additional identifying restriction. A standard approach to deal with the identification problem is to instrument for the behavior of the sibling. While sound, it has been difficult for researchers to find instrumental variable candidates with observational data.

Abstracting from sibling interaction effects, there is an active literature using sibling and twin difference regressions to estimate the effect of own covariates on behavior. These sibling difference regressions have been used to study the effects of birth order, gender, birth weight, neighborhoods, family structure, parental employment, age of immigration, etc. on a child’s schooling, earnings or other outcomes. Explicitly or implicitly, this literature makes two assumptions: (1) Sibling differences in the covariates are orthogonal to the unobserved common family factors. (2) Siblings’ covariates do not directly affect own behavior. With these two assumptions and the assumed absence of sibling interaction effects, sibling difference regressions are used to obtain consistent estimates of the direct effects of own covariates on behavior.

Building on sibling difference regressions, this paper considers a new instrumental variable (IV) strategy for estimating sibling interaction effects in the absence of variables which are uncorrelated with the unobserved family factors to use as instrumental variable candidates. In addition to assumptions (1) and (2) above, we add assumption (3): Standardized direct responses to variations in own covariates differ for different observable types of siblings. With these three assumptions, we construct instrumental variable candidates from covariates that are correlated to unobserved common family factors. With at least two such covariates affecting individual behavior, the sibling interaction effects model is identified.

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1E.g. Behrman 1997; Buhrmester 1992; Rogers and Rowe 1988; Urdan, et. al. 2007; Widmer 1997.
3Blume, et. al. 2010 surveys different methods developed to estimate peer interaction effects since Manski.
4E.g. Oettinger 2000.
5“It is hard to see how, in typical socioeconomic contexts, such instruments may be found, since the instruments must be known on a priori grounds to be uncorrelated with both the undertheorized group effect and idiosyncratic effect.” (Blume, et. al. 2010). Barrera-Osorio, et. al. 2008; Ferreira, et. al. 2009 use field experiments to create such instruments.
7A careful statement of the methodology is in Ermish, Francesconi and Pechalín 2004.
Following the sibling difference literature, our assumptions (1) and (2) are maintained and untestable. Assumption (3) is testable. If it fails or the differences in direct responses are not large, we will run into the weak instruments problem under our IV estimation strategy. The standard linear interaction effects model with common direct response parameters across different observable types of siblings violates assumption (3). Relaxing the common direct response parameters assumption aids identification because it generates more distinct observable moment conditions than additional parameters, and with two or more such covariates, enough for identification. Thus one can test whether sibling interaction effects are identified in our framework.

The paper presents two extensions. First, the method is extended to the case where the interaction effects depend on own type as well as the type of the sibling. In this case, we require that the direct responses only depend on own type. Second, the method is also extended to the case where some of the covariates are themselves subject to interaction influences.

In this paper, we study sibling interaction effects in long run wages between siblings by gender. We focus on gender interaction effects because the vast literature on gender differences in long run wages (wages hereafter) have established that some covariates such as marital status and fertility affect female and male wages differently (E.g. Blau and Kahn 2000; Ginther and Zavodny 2001; Korenman and Neumark 1991; Waldfogel 1998). We exploit this a priori established gender differences in direct effects of these covariates on wages to estimate sibling wage interaction effects which depend on own type as well as the type of the sibling. We will also study birth order sibling interaction effects in passing.8

The second part of this paper estimates the model with data from the 1979 National Longitudinal Survey of Youth (79 NLSY) data set.

There are few studies which estimate sibling interaction effects using observational data.9 Oettinger 2000 studies sibling interaction effects in schooling by birth order. He assumes the availability of instruments that are uncorrelated with the unobserved common family factor. Also using the 79 NLSY, he finds that the schooling attainment of older siblings affect the younger siblings but not vice versa. Another paper is Altonji, Cattan and Ware 2010. They use a dynamic correlated random effects model with time varying family effects to study sibling peer effects in risky behaviors such as smoking, drinking and drug use in the 79 NLSY. They assume that initially, an older sibling can influence a younger sibling but not vice versa for identification. They find evidence for weak sibling peer effects for risky behaviors. Kuziemko 2006 studied sibling peer effect on the timing of fertility using data from both the Panel Study of Income Dynamics and NLSY. She found that a woman is more likely to have a child within two years of her sister having a child.

8Researchers have found birth order effects in schooling attainment (E.g. Black, et. al. 2005; Behrman and Taubman 1986; Kantarevic and Mechoolan 2006). 9Recent studies use field experiments to estimate the sibling peer effects in schooling attainment (E.g. Barrera-Osorio, et. al. 2008; Ferreira 2009).
1 The identification problem in estimating sibling interaction effects

This section sketches the basic identification problem of estimating sibling interaction effects in the linear interaction effects model for long run wages using observational data without covariates. This identification problem is well known and our discussion is brief.

Consider a random sample of \( h = 1, \ldots, H \) families. \( i = 1 \) and \( i = 2 \) denote the two children. The long run log wage (wage for short) of child \( i \) with sibling \( j \) in family \( h \) is:

\[
W_{ih} = \pi_i(W_{jh} - W_{ih}) + \eta_h + \varepsilon_{ih}
\]
\[
\text{var}(\varepsilon_{ih}) = \sigma^2_{\varepsilon_1}, \quad \text{var}(\eta_h) = \sigma^2_{\eta}
\]
\[
\varepsilon_{1h} \perp \varepsilon_{2h}; \varepsilon_{ih} \perp \eta
\]

\( \eta_h \) is the unobserved common family effect. \( \varepsilon_{ih} \) are indiosyncratic individual effects. The assumption on the covariance structure of the unobservables are standard in the correlated random effects model.

\( \pi_1 \) and \( \pi_2 \) are the sibling interaction effects coefficients. They can be due to sibling peer effects, family specific capital market constraints, parental preferences and actions, and so on.\(^{10}\)

Since it is difficult to interpret the quantitative significance of \( \pi_1 \) and \( \pi_2 \), we consider another measure of interaction contribution, \( P \):

\[
0 \leq P = 1 - \frac{\sigma^2_{ij}}{\text{cov}(W_i, W_j)} \leq 1
\]

\( P \) is the interaction effects contribution to \( \text{cov}(W_i, W_j) \). \( P \) lies between 0 and 1. If \( P \) is zero, it says that all the covariance between \( W_{1h} \) and \( W_{2h} \) is due to the unobserved common family factor \( \eta_h \). If \( P \) is 1, it says that all the covariance between \( W_{1h} \) and \( W_{2h} \) is due to sibling interaction effects.

The above model has five unknown parameters: \( \pi_1, \pi_2, \sigma^2_{\varepsilon_1}, \sigma^2_{\varepsilon_2}, \) and \( \sigma^2_{\eta} \). There are three observable moments: \( \text{var}(W_1) \), \( \text{var}(W_2) \) and \( \text{cov}(W_1, W_2) \). The interaction coefficients \( \pi_1 \) and \( \pi_2 \), and \( \sigma^2_{\eta} \) are unidentified.

The model sketched above is more general than the standard linear interaction effect model which assumes common parameters across siblings. Let \( \pi_1 = \pi_2 = \pi, \sigma^2_{\varepsilon_1} = \sigma^2_{\varepsilon_2} = \sigma^2_{\varepsilon} \). Then \( \sigma^2_{W_1} = \sigma^2_{W_2} \). The system reduces to two distinct observable moments and three parameters. \( \sigma^2_{\eta} \) and the interaction coefficient \( \pi \) remain unidentified. \( P \) remains unidentified.

2 Adding covariates to the wage equation

We expand the above model to include a $k \times 1$ vector of own covariates, $Q_{ih}$, for each child $i$ in family $h$. The covariates are equal to a $k \times 1$ vector of unobserved common family components, $C_h$, and a $k \times 1$ vector of idiosyncratic components, $\Omega_{ih}$. Let $E$ be the expectations operator. Then:

\begin{align*}
Q_{ih} &= C_h + \Omega_{ih} \quad (1) \\
W_{ih} &= \pi_i(W_{jh} - W_{ih}) + Q_{ih}' \beta_{Qi} + v_{ih} \quad (2) \\
v_{ih} &= C_h' \beta_C + \eta_h + \varepsilon_{ih} \quad (3) \\
C_h &\perp \Omega_{ih}; C_h, \Omega_{ih} \perp \eta_h, \varepsilon_{ih}; \Omega_{ih} \perp \Omega_{jh}; \varepsilon_{ih} \perp \Omega_{ih} \\
E(C_h) &= C, \text{cov}(C_h) = \Sigma_C, E(\Omega_{ih}) = 0, \text{cov}(\Omega_{ih}) = \Sigma_{\Omega_{ih}} \\
E(\eta_h) &= \eta, \text{cov}(\eta_h) = \sigma_{\eta_i}^2, E(\varepsilon_{ih}) = 0, \text{cov}(\varepsilon_{ih}) = \sigma_{\varepsilon_i}^2
\end{align*}

The above wage equation, (2) expresses the interaction effect $\pi_i$ as depending on the difference between the siblings’ wages. We put it in the standard linear-in-mean peer effect form below in equation (6).

Each child’s wage equation has been extended to include own covariates, $Q_{ih}$, and also to allow the common family effects $C_h$ to appear in the schooling equation. We further add an additional common family effect, $\eta_h$, that is orthogonal to $C_h$. One interpretation of $\eta_h$ is that it is an unobserved family effect which is realized after $C_h$ are realized. The assumption $\Omega_{ih} \perp \varepsilon_{ih}$ means that we are ruling out unobserved individual effects in own covariates which are correlated and the unobserved idiosyncratic factors in own wages. This is assumption (1) which is standard in siblings differences studies.

Note that $\pi_i$ is type dependent. We allow the sibling interaction effect to depend on the type of the sibling.

For readers familiar with Manski’s 1993 setup, we do not allow contextual effects in the above model. This means that we do not allow the sibling’s covariates to affect own wages directly.\footnote{In section 5, we allow a sibling’s covariate to directly affect an own covariate.} This is a necessary exclusion restriction in the sibling difference regressions literature (our assumption (2)). Without this exclusion restriction, even absent sibling interaction effects, the casual effects of own covariates, such as birth weight, on own wages cannot be estimated from a sibling difference regression. Put another way, we are extending the sibling difference framework, which maintains assumptions (1) and (2), to allow for potential sibling interaction effects.

As is well known and shown in the appendix for convenience, equation (2) can be derived from a unitary model of the family or as $i$’s reaction function to sibling $j$. That is, the above interaction model may also be interpreted as the equilibrium outcome of a static non-cooperative game between two siblings. Thus our empirical framework sheds no light on whether the family is acting efficiently or not.
\[ P_k = 1 - \frac{\sigma^2 + (\beta_C + \beta_Q_i)\Sigma_C(\beta_C + \beta_Q_j)}{\sigma_{W_i,W_j}} \]

\[ \sigma^2 + (\beta_C + \beta_Q_i)\Sigma_C(\beta_C + \beta_Q_j) = \sigma_{W_i,W_j} \text{ if } \pi_i = 0. \] At the other extreme, if \( \sigma^2 = 0 \) and \( \Sigma_C = 0 \), and there is no direct common effect, then \( P_k = 1 \) which implies that all the covariation in wages are due to sibling interaction effects.

### 3 An IV estimation strategy

From the wage equation (2), we get the sibling difference equation:

\[
W_{ih} - W_{jh} = \frac{\beta Q_1}{\Pi} Q_{1h} - \frac{\beta Q_2}{\Pi} Q_{2h} + \frac{\varepsilon_{ih} - \varepsilon_{jh}}{\Pi} \tag{4}
\]

\[
\Pi = 1 + \pi_1 + \pi_2 \tag{5}
\]

By assumption, the covariates, \( Q_{1h} \) and \( Q_{2h} \), are orthogonal to the error term, \( \frac{\varepsilon_{ih} - \varepsilon_{jh}}{\Pi} \). So we can estimate equation (4) by OLS to obtain consistent estimates of the vectors \( \frac{\beta Q_1}{\Pi} \) and \( \frac{\beta Q_2}{\Pi} \). We will use these estimates momentarily.

Rewrite the wage equation (2) in the standard linear peer effect form:

\[
W_{ih} = \frac{\pi_i}{1 + \pi_i} W_{jh} + \left( \frac{\beta Q_i}{1 + \pi_i} \right)' Q_{ih} + \frac{v_{ih}}{1 + \pi_i} \tag{6}
\]

Considering it as a regression equation, it has \( k + 1 \) covariates, \( W_{jh} \) and \( Q_{ih} \). We cannot consistently estimate the parameters consistently with OLS because the covariates are correlated with the error term of the regression.

From equation (1), we observe that

\[
Q_{ih} - Q_{jh} = \Delta Q_h = \Delta \Omega_h
\]

\( \Delta Q_h \) is a \( k \times 1 \) vector and it is uncorrelated with the error term in the wage equation, (6). So it can serve as \( k \) instrumental variable candidates for estimating the wage equation by IV. However as we noted, the wage equation has \( k + 1 \) covariates. So this direct estimation strategy is short one instrument.

First, let one covariate, \( q^i_{ih} \), not have a family effect, i.e. \( c^i_h = 0 \). Then \( q^i_{ih} \) can be its own instrument and \( q^j_{jh} \) can be an instrument for \( W_{ih} \). In this case, the standard IV estimation strategy applies and and one can obtain consistent estimates of \( \pi_i \) independent of assumption (3).

Now, consider the case where \( C_h \neq 0 \). Then one cannot use the standard IV estimation strategy as described in the previous paragraph.

\[ ^{12} \text{E.g. Barrera-Osorio, et. al. 2008; Ferreira, et. al. 2009; Oettinger 2000.} \]
In this case, consider rewriting the wage equation (6) as:

\[ W_{ih} = \frac{\pi_i}{1 + \pi_i} W_{jh} + (1 + \frac{\pi_j}{1 + \pi_i}) \tilde{Q}_{ih} + \frac{v_{ih}}{1 + \pi_i} \]  

(7)

\[ \tilde{Q}_{ih} = \frac{\beta_{Q_{ih}}}{\Pi} Q_{ih} \]  

(8)

We have reduced the wage equation (6) with \( k + 1 \) covariates to an equation with two covariates, \( W_{jh} \) and \( \tilde{Q}_{ih} \). If we can construct \( \tilde{Q}_{ih} \), the \( k \times 1 \) vector \( \Delta Q_h \) can serve as instrumental variable candidates for \( W_{jh} \) and \( \tilde{Q}_{ih} \). With consistent estimates of \( \Sigma_{\Omega j} = E(\Omega_{jh}\Omega_{jh}') \) for \( j = 1, 2 \). Then:

**Proposition 1** \( \pi_1 \) and \( \pi_2 \) are identified if and only if for any ordered pair \( (d_1, d_2) \neq (0, 0) \),

\[ d_1 \cdot \Sigma_{\Omega_1} \beta_{Q_1} + d_2 \cdot \Sigma_{\Omega_2} \beta_{Q_2} \neq 0 \]  

(9)

The proof is in the end of this section.

The above restriction (9) is the assumption (3) of this paper: The standardized direct responses to variations in own covariates must differ across different types of siblings.

When \( k \geq 2 \), identification obtains generically. Identification would fail on a set of measure zero. Of particular significance is the case of the linear interaction effects model with common parameters:

**Corollary 2** \( \pi_1 \) and \( \pi_2 \) are unidentified in the case of the linear interaction effects model with common parameters, \( \Sigma_{Q_1} = \Sigma_{Q_2} \) and \( \beta_{Q_1} = \beta_{Q_2} \).

If \( \pi_1 = \pi_2 = \pi \), equation (7) can be rewritten as:

\[ W_{1h} - \tilde{Q}_{2h} = \pi(W_{2h} - W_{1h} + 2\tilde{Q}_{1h}) + v_{ih} \]  

(10)

The above equation has one covariate, \( (W_{2h} - W_{1h} + 2\tilde{Q}_{1h}) \), which is correlated with \( v_{1h} \). With \( k \) instruments, \( \Delta Q \), we can estimate \( \pi \) by IV. Similarly,

**Corollary 3** Let \( k \geq 1 \) and restriction (9) applies. Let \( \pi_1 = \pi_2 \) or \( \pi_2 = 0 \). \( \pi_1 \) is identified.

### 3.1

#### 3.2 Proof of proposition:

Let \( Z = E(\Delta Q_h (W_{jh} \quad \tilde{Q}_{ih})) \) where \( Z \in \mathbb{R}^{k \times 2} \).

The rank condition for identification requires

\[ \text{rank}(Z) = 2 \]
The first column of $Z$ is
\[
E(\Delta Q_h, W_{jh}^h) = E\left(\Delta \Omega_h \left(Q'_{2h} \frac{\beta Q_2}{\Pi}\right)\right) = \Sigma_{\Omega2} \frac{\beta Q_2}{\Pi}
\]

The second column of $Z$ is
\[
E(\Delta Q_h, \tilde{Q}_{ih}) = E\left(\Delta \Omega_h \left(Q'_{1h} \frac{\beta Q_1}{\Pi}\right)\right) = \Sigma_{\Omega1} \frac{\beta Q_1}{\Pi}
\]

Therefore $Z$ has rank $< 2$ iff there exists an ordered pair $(d_1, d_2) \neq (0, 0)$ such that
\[
\left(\Sigma_{\Omega1} \frac{\beta Q_1}{\Pi}\right) d_1 + \left(\Sigma_{\Omega2} \frac{\beta Q_2}{\Pi}\right) d_2 = 0
\]
or equivalently,
\[
d_1 \cdot \Sigma_{\Omega1} \beta Q_1 + d_2 \cdot \Sigma_{\Omega2} \beta Q_2 = 0
\]

\[\blacksquare\]

4 Estimating interaction effects which depend on own and sibling’s types

The above model assumes that the sibling interaction effects depend only on own type. When the type of a child is the gender of the child, there are theories and evidence which suggest that the interaction effects may depend on both the own type and the type of the sibling.\footnote{E.g. Butcher and Case 1994; Hauser and Kuo 1998; Keestner 1997; Kuziemko 2006; Steelman, et. al. 2002.} In this section, we extend our framework to this case. Let $\pi_{ij}$ be the response of child $i$’s wages to sibling $j$’s wages where the type of an individual is his or her gender, $i, j \in \{m, f\}$, where $m$ denotes male and $f$ denotes female. Since a boy may respond differently to a brother’s or a sister’s wage and vice versa, there are four interaction effects to be estimated:

$\pi_{mm}, \pi_{mf}, \pi_{fm}, \pi_{ff}$

Assume that the direct effects, $\beta Q_m$ and $\beta Q_f$, only depend on the gender of the child.

Then child $i$’s covariates and wage in family $h$ are:

\[
Q_{ih} = C_h + \omega_{ih}
\]
\[
W_{ih} = \pi_{ij}(W_{jh} - W_{ih}) + Q_{ih}' \beta_{Qi} + C_h' \gamma_{iC} + \phi_i \eta_h + \epsilon_{ih}
\]
\[ \gamma_{iC} \text{ and } \phi_i \text{ allows the family factors in wages to depend on the gender of child } i. \Sigma_{\delta m} \text{ and } \Sigma_{\delta f} \text{ also depend on gender of the child. Unlike the previous section, we relax the assumption that the family factors in wages must be independent of the type of the child. The reason why we can relax that assumption is that we are able to observe families with two children of the same gender and families with mixed gender children. If the type of a child is birth order, then it is not possible to have two children in a family with the same birth order.} \]

Following earlier definitions, \( P_{ij} \), the contribution of the interaction effect to the covariance of the siblings long run wages, \( \sigma_{W_1,W_2} \), is:

\[ P_{ij} = 1 - \frac{(\gamma_{iC} + \beta_{Qi})' \Sigma_C (\gamma_{jC} + \beta_{Qj}) + \phi_i \phi_j \sigma_y^2}{\sigma_{W_1,W_2}} \]

Then:

**Corollary 4** The model is identified for \( k \geq 2 \) and if there does not exist a scalar \( \lambda \neq 0 \) such that:

\[ \Sigma_{\delta m} \beta_{Qm} \neq \lambda \Sigma_{\delta f} \beta_{Qf} \]

A sketch of the proof is as follows. First, consider the subsample of families with same gendered siblings, \( i = g \) and \( j = g \). The sibling difference regression becomes:

\[ \Delta W_{gh} = \frac{\gamma_{Qg}}{\Pi_{gg}} \Delta Q_{gh} + \frac{\Delta \varepsilon_{gh}}{\Pi_{gg}} \]

\[ \Pi_{gg} = 1 + 2\gamma_{gg} \]

From the above regressions for \( \{m,m\} \) and \( \{f,f\} \) families, we can get consistent estimates of \( \frac{\beta_{Qm}}{\Pi_{mm}} \) and \( \frac{\beta_{Qf}}{\Pi_{ff}} \).

Then from the mixed gender families, \( \{m,f\} \), the wage equations of the children are:

\[ W_{mh} = \frac{\gamma_{mf}}{1 + \gamma_{mf}} W_{fh} + \frac{\Pi_{mm}}{1 + \gamma_{mf}} \tilde{Q}_{mh} + v_{mh} \quad (11) \]

\[ W_{fh} = \frac{\gamma_{fm}}{1 + \gamma_{fm}} W_{mh} + \frac{\Pi_{mm}}{1 + \gamma_{fm}} \tilde{Q}_{fh} + v_{fh} \quad (12) \]

\[ \tilde{Q}_{mh} = \frac{Q_{mh} \beta_{Qm}}{\Pi_{mm}} \quad \tilde{Q}_{fh} = \frac{Q_{fh} \beta_{Qf}}{\Pi_{ff}} \]

\[ v_{gh} = C_h \gamma_{gC} + \phi_g \eta_h + \varepsilon_{gh} \]

We can estimate equations (11) by IV using \( \Delta Q_{mf} \) as instruments and obtain consistent estimates of \( \tau_{mf} \) and \( \Pi_{mm} (\pi_{mm}) \). We can also estimate equations (12) by IV also using \( \Delta Q_{mf} \) as instruments and obtain consistent estimates of \( \tau_{fm} \) and \( \Pi_{ff} (\pi_{ff}) \).
So in the case of gender interaction effects where there are four interaction effects and gender specific effects of family factors in wages, we still only need \( k \geq 2 \) covariates. We are restricting the direct effects to be independent of the gender of the sibling; and the family effects in the covariates to be gender independent.

5 Identification when covariates are subject to interaction effects

The section deals with the case where some of the covariates in an interaction effects model are subject to interaction influences themselves. For convenience, we return to the case where the interaction effect only depends on own type. Returning to the wage model (1) to (2), let \( q_{ih}^s \), an element of \( Q_{ih} \) be the schooling attainment of child \( i \) where \( q_{ih}^s \) may itself be subject to sibling interaction effects:

\[
q_{ih}^s = \gamma_i(q_{jh}^s - q_{ih}^s) + c_i^s + \omega_{ih}^s
\]  

(13)

In the previous sections, \( \gamma_i = 0 \). Here we assume \( \gamma_i \neq 0 \). Recall \( \Pi = 1 + \pi_1 + \pi_2 \) and as before, we can construct:

\[
\Delta W_h = \frac{\beta_1}{\Pi} Q_{1h} - \frac{\beta_2}{\Pi} Q_{2h} + \frac{\Delta v_h}{\Pi}
\]  

(14)

\[
W_{ih} = \frac{1}{1 + \pi_i} W_{jh} + (1 + \frac{\pi_j}{1 + \pi_i}) \tilde{Q}_{ih} + \frac{v_{ih}}{1 + \pi_i}
\]  

(15)

\[
v_{ih} = C_{ih}^s \beta C + \eta_h + \varepsilon_{ih}
\]  

(16)

\[
\tilde{Q}_{ih} = \frac{\beta_1}{\Pi} Q_{ih}
\]  

(17)

As before, we can estimate equation (14) by OLS to get consistent estimates of \( \frac{\beta_1}{\Pi} \) and \( \frac{\beta_2}{\Pi} \).

Then to estimate equation (15) by IV, we need instruments for \( W_{jh} \) and \( \tilde{Q}_{ih} \). \( \Delta Q_{ih} \) will serve as \( k - 1 \) instruments. From equation (13),

\[
\Delta q_{ih}^s = \frac{\Delta \omega_{ih}^s}{1 + \gamma_1 + \gamma_2}
\]

which is uncorrelated with the unobserved family factors in (16). Thus we can also use \( \Delta q_{ih}^s \) as an additional instrument. Thus, as long as \( k \geq 2 \), we have enough instruments even if some of the covariates in the wage equations are subject to interaction influences themselves.

So although we do not allow a sibling’s covariate to have a direct effect on own wages, we do allow a sibling’s covariate to have a direct effect on an own covariate. From a behavioral point of view, this is a significant relaxation of our basic model.
6 Empirical results

References


Appendix: Two almost observationally equivalent behavioral static models

This section provides two almost observationally equivalent behavioral static models of sibling interaction effects in earnings (wage) attainment which underly the empirical framework. Except for a special case where the younger child’s earnings does not affect the cost of the older’s child earnings attainment, the static unitary model of earnings attainment cannot be distinguished from the non-cooperative model of earnings attainment. Second, except for the special case, there is no apriori restriction on the signs of \( \pi_i \). They depend positively on the degree of inequality aversion of within family earnings differences, positively on sibling peer effects, and negatively on capital market restrictions on family borrowing for the children’s earnings attainment.

First, consider a static unitary model of the family. Let \( W_{1h} \) and \( W_{2h} \) be the earnings attainments of the older and younger sibling in family \( h \) respectively. The objective function of the family is:

\[
U^h(W_{1h}, W_{2h}) = B^h(W_{1h}, W_{2h}) - C^h(W_{1h}, W_{2h})
\]

where \( B^h(W_{1h}, W_{2h}) \) is the benefit of earnings and \( C^h(W_{1h}, W_{2h}) \) is the cost of earnings attainment.

Assuming quadratic benefit and cost functions, let

\[
B^h(W_{1h}, W_{2h}) = b_{1h}W_{1h} + b_{2h}W_{2h} - \frac{b_{12}}{2}W_{1h}^2 - \frac{b_{22}}{2}W_{2h}^2 - b_3(W_{1h} - W_{2h})^2; \quad b_{1h}, b_{2h}, b_{12}, b_{22}, b_3 > 0
\]

\[
C^h(W_{1h}, W_{2h}) = (c_{1h} - d_1W_{2h})W_{1h} + (c_{2h} - d_2W_{1h})W_{2h} + \frac{c_{12}}{2}W_{1h}^2 + \frac{c_{22}}{2}W_{2h}^2;
\]

\[
c_{1h} - d_1\overline{W}, c_{2h} - d_2\overline{W}, c_{12}, c_{22}, c_3, d_1, d_2 > 0
\]

where \( \overline{W} \) is maximum earnings attainment.

In addition to diminishing marginal benefit of earnings, the family also suffers from inequality aversion of within family earnings attainments when \( b_3 > 0 \).

\( d_1 > 0 \) and \( d_2 > 0 \) imply that the marginal cost of earnings for one sibling is falling in the earnings attainment of the other. \( b_3 > 0 \), \( d_1 > 0 \) and \( d_2 > 0 \) all lead to positive empirical sibling interaction effects in earnings attainment.

On the other hand, capital market constraints on household borrowing will reduce \( d_1 \) and \( d_2 \). If these constraints sufficiently bind, \( d_1 \) and or \( d_2 \) may be negative. I.e. if one sibling acquires a lot of earnings, it increases the marginal cost of earnings for the other child. So borrowing constraints can lead to negative empirical sibling interaction effects in earnings attainment.

Substituting \( B^h(W_{1h}, W_{2h}) \) and \( C^h(W_{1h}, W_{2h}) \) into the objective function of the family (18) will generate the reduced form objective function:
\[ U^h(W_{1h}, W_{2h}) = \alpha_{1h}W_{1h} + \alpha_{2h}W_{2h} - \frac{\alpha_{11}^2}{2}W_{1}^2 - \frac{\alpha_{22}^2}{2}W_{2}^2 + \alpha_3W_1W_2 \]  
\( \alpha_{1h} - \alpha_{12}, \alpha_{2h} - \alpha_{22}, \alpha_{12} - \alpha_3, \alpha_{22} - \alpha_3, \alpha_3 > 0 \) \hspace{1cm} (19)

The sibling interaction effect in earnings is captured by \( \alpha_3 = 2b_3 - d_1 - d_2 \).

If \( \alpha_3 > 0 \), \( W_{1h} \) and \( W_{2h} \) are complements. If the family increases \( W_{1h} \), it will increase the marginal value of increasing \( W_{2h} \) and vice versa. If \( \alpha_3 < 0 \), \( W_{1h} \) and \( W_{2h} \) are substitutes. If the family increases \( W_{1h} \), it will increase the marginal cost of increasing \( W_{2h} \) and vice versa.

The family chooses \( W_{1h} \) and \( W_{2h} \) to maximize the above reduced form objective function (19). The optimal choices of \( W_{1h} \) and \( W_{2h} \) will satisfy:

\[ W_{ih} = \frac{\alpha_{ih}}{\alpha_{i,j} - \alpha_3} + \frac{\alpha_3}{\alpha_{i,j} - \alpha_3}(W_{jh} - W_{ih}); i = 1, 2 \] 
\hspace{1cm} (20)

Let

\[ \frac{\alpha_{ih}}{\alpha_{i,j} - \alpha_3} = \beta_{i0} + \beta_{Q,i}Q_{ih} + \beta_{C,i}C_{ih} + \eta_i + \varepsilon_{ih}; i = 1, 2 \] 
\hspace{1cm} (21)

\[ \frac{\alpha_3}{\alpha_{i,j} - \alpha_3} = \pi_i; i = 1, 2 \] 
\hspace{1cm} (22)

Then we obtain the empirical sibling earnings equations:

\[ W_{ih} = \pi_i(W_{jh} - W_{ih}) + \beta_{Q,i}Q_{ih} + \beta_{C,i}C_{ih} + \eta_i + \varepsilon_{ih}; i = 1, 2 \] 
\hspace{1cm} (23)

The point of the behavioral model is to show that \( \pi_1 \) and \( \beta_{S2} \) encapsulate peer effects, capital market constraints as well as inequality aversion in within family outcomes.\(^{14}\)

While \( \pi_1 \) do not have to equal \( \pi_2 \), we cannot have \( \pi_1 = 0 \) and \( \pi_2 \neq 0 \). If the younger sibling’s earnings does not affect the older sibling’s earnings, then the older’s child education cannot affect the younger one either. This is true even if \( d_1 = 0 \) where the younger child’s earnings attainment do not affect the marginal cost of the older child’s earnings, and or \( b_3 = 0 \) where the family does not suffer from inequality aversion in earnings differences. As long as \( d_2 \neq 0 \)

\(^{14}\)Instead of (21), consider:

\[ \alpha_{if} = \beta_0 + \beta_{Q,i}Q_{if} + \beta_{C,i}C_{if} + \eta_f + \varepsilon_{if} \] 
\hspace{1cm} (24)

Then instead of (23), we have the empirical sibling schooling equations:

\[ S_{if} = \frac{\alpha_3}{\alpha_{i,\neq i} - \alpha_3}(S_{\neq i,f} - S_{if}) + \frac{\beta_{Q,i}}{\alpha_{i,\neq i} - \alpha_3}Q_{if} + \frac{\beta_{C,i}}{\alpha_{i,\neq i} - \alpha_3}C_{if} + \frac{\eta_f}{\alpha_{i,\neq i} - \alpha_3} + \frac{\varepsilon_{if}}{\alpha_{i,\neq i} - \alpha_3}; i = 1, 2 \] 
\hspace{1cm} (25)

This model also has 11 unknowns:

\[ \sigma_{\varepsilon_{11}}^2, \sigma_{\varepsilon_{21}}^2, \sigma_{\varepsilon_{22}}^2, \alpha_3, \alpha_{12}, \alpha_{22}, \beta_{Q,i}, \beta_{C,i}, \sigma_{\varepsilon_{11}}^2, \sigma_{\varepsilon_{22}}^2, \eta_f \]

Thus in terms of identification, it is not an improvement over the previous empirical model.
which implies that increasing the older’s child earnings will affect the marginal
cost of the younger’s child earnings, the family will choose the older child’s
earnings internalizing this “externality”. Put another way, if \( d_2 > 0 \) and the
family wants to increase \( S_{2h} \), it can do more cheaply by first increasing \( S_{1h} \).

In order to have \( \pi_1 = 0 \) and \( \pi_2 \neq 0 \), we have to relax the unitary model.
Consider a non-cooperative model model of siblings’ earnings attainment. Let
the payoff from earnings for sibling \( i \) be:

\[
P^{ih}(W_{ih}) = (p_0^{ih} + p_1^{ih}W_{jh})W_{ih} - \frac{p_2^{ih}}{2}W_{ih}^2; \quad i = 1, 2
\]

The equilibrium earnings reaction functions satisfy:

\[
W_{ih} = \frac{p_0^{ih}}{p_2^{ih} - p_1^{ih}} + \frac{p_1^{ih}}{p_2^{ih} - p_1^{ih}}(W_{jh} - W_{ih}); \quad i = 1, 2
\]

Let

\[
\frac{p_0^{ih}}{p_2^{ih} - p_1^{ih}} = \beta_{i0} + \beta_{Qi}Q_{ih} + \beta_{iC}C_{ih} + \eta_{ih} + \varepsilon_{ih}; \quad i = 1, 2
\]

\[
\frac{p_1^{ih}}{p_2^{ih} - p_1^{ih}} = \pi_i; \quad i = 1, 2
\]

Then empirical earnings attainment equations are:

\[
W_{ih} = \pi_i(W_{jh} - W_{ih}) + \beta_{Qi}Q_{ih} + \beta_{iC}C_{ih} + \eta_{ih} + \varepsilon_{ih}; \quad i = 1, 2
\]

If \( p_1^{ih} = 0 \), the younger child’s earnings does not affect the cost of the older’s
child earnings. Let \( p_1^{ih} = 0 \) and \( p_2^{ih} \neq 0 \). Then \( \pi_1 = 0 \) and \( \pi_2 \neq 0 \). Unlike the
unitary model, the older child makes his own earnings decision without taking
into account how his decision affect’s his sibling’s cost of earnings.

Except for the special case of \( \pi_1 = 0 \) and \( \pi_2 \neq 0 \), the static unitary model
and the non-cooperative model of sibling earnings attainment cannot be distin-
guished. Following Oettinger, a sequential unitary model can rationalize \( \pi_1 = 0 \).
Consider modifying the unitary model discussed above as follows. Assume that
parents have to choose \( W_{1h} \) before \( Q_{2h} \) and \( \varepsilon_{2h} \) are realized. The parents’ choice
of \( W_{1h} \) will be independent of \( f_{2h} \) and \( \varepsilon_{2h} \). In this case, \( \pi_1 = 0 \).